Business Research Methods

Course Code – MGMT4013

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Content

- **Hypotheses testing :**
  - **t test**
  - **z test**
  - **Chi-square test**
  - **Analysis of Variance (ANOVA)**

❖ Hypothesis Testing

A hypothesis test is a formal way to make a decision based on statistical analysis. A hypothesis test has the following general steps:

- Set up two contradictory hypotheses. One represents our "assumption".
- Perform an experiment to collect data. Analyze the data using the appropriate distribution.
- Decide if the experimental data contradicts the assumption or not.
- Translate the decision into a clear, non-technical conclusion.

❖ Null and Alternative Hypotheses

Hypothesis tests are tests about a population parameter (μ or p). We will do hypothesis tests about population mean and population proportion p.

The null hypothesis (H0) is a statement involving equality (=; $\leq$;$\geq$) about a population parameter. We assume the null hypothesis is true to do our analysis.

The alternative hypothesis (Ha) is a statement that contradicts the null hypothesis. The alternative hypothesis is what we conclude is true if the experimental results lead us to conclude that the null hypothesis (our assumption) is false.

The alternative hypothesis must not involve equality ($\neq$; <;>).

The exact statement of the null and alternative hypotheses depend on the claim that you are testing.

❖ Outcomes and the Type I and Type II Errors

Hypothesis tests are based on incomplete information, since a sample can never give us complete information about a population. Therefore, there is always a chance that our conclusion has been made in error.

There are two possible types of error:

The first possible error is if we conclude that the null hypothesis (our assumption) is invalid (choosing to believe the alternative hypothesis), when the null hypothesis is really true. This is called a Type I error.

Type I error = { Deciding to reject the null when the null is true
incorrectly supporting the alternative

The other possible error is if we conclude that the null hypothesis (our assumption) seems reasonable (choosing not to believe the alternative hypothesis), when the null hypothesis is really false. This is called a Type II error.

Type II error = { Failing to reject the null when the null is False
incorrectly NOT supporting the alternative

TYPE I and TYPE II ERROR IN TABULAR FORM

	Decision	
	Accept H0	**Reject H0**
H0 True	Correct decision	**Type I Error**
H0 False	**Type II Error**	Correct Decision

When a Null hypothesis is tested, there may be four possible outcome:

I. The Null Hypothesis is true but our test rejects it.
II. The Null Hypothesis is false but our test accept it.
III. The Null Hypothesis is true and our test accepts it.
IV. The Null Hypothesis is false but our test rejects it.

Type I Error : Rejecting Null Hypothesis when Null Hypothesis is true. It is called ‘α-error’.

Type II Error : Accepting Null Hypothesis when Null Hypothesis is false. It is called ‘β-error’.

❖ Outcomes and the Type I and Type II Errors Cont…

It is important to be aware of the probability of getting each type of error. The following notation is used:

$$\alpha = \begin{cases} \text{P(Type I error)} \\ \text{P(decide to reject null} \mid \text{null is true)} \\ \text{P(incorrectly supporting alternative)} \\ \text{the } \textbf{significance level} \text{ of the test} \end{cases}$$

$$\beta = \begin{cases} \text{P(Type II error)} \\ \text{P(decide to ``accept'' null} \mid \text{null is false)} \\ \text{P(incorrectly NOT supporting alternative)} \end{cases}$$

$$1 - \beta = \begin{cases} \text{P(decide to reject null} \mid \text{null is false)} \\ \text{P(correctly supporting the alternative)} \\ \text{the } \textbf{power} \text{ of the test} \end{cases}$$

❖ Outcomes and the Type I and Type II Errors Cont…

The signicance level α is the probability that we incorrectly reject the assumption (null) and support the alternative hypothesis. In practice, a data scientist chooses the signicance level based on the severity of the consequence of incorrectly supporting the alternative. In our problems, the signicance level will be provided.

The **power** of a test is the probability of correctly supporting a true alternative hypothesis. Usually we are testing if there is statistical evidence to support a claim represented by the alternative hypothesis. The power of the test tells us how often we “get it right” when the claim is true.

❖ Distribution Needed for Hypothesis Testing

The sample statistic (the best point estimate for the population parameter, which we use to decide whether or not to reject the null hypothesis) and distribution for hypothesis tests are basically the same as for confidence intervals.

The only difference is that for hypothesis tests, we assume that the population mean (or population proportion) is known: it is the value supplied by the null hypothesis. (This is how we \assume the null hypothesis is true" when we are testing if our sample data contradicts our assumption.)

When **testing a claim about population mean** μ, ONE of the following two requirements must be met, so that the Central Limit Theorem applies and we can assume the random variable, $\overline{x}$ is normally distributed:

- The sample size must be relatively large (many books recommend at least 30 samples), OR
- the sample appears to come from a normally distributed population.

It is very important to verify these requirements in real life. In the problems we are usually told to assume the second condition holds if the sample size is small.

❖ Stating Hypotheses

The first step in conducting a test of statistical significance is to state the hypotheses.

A significance test starts with a careful statement of the claims being compared. The claim tested by a statistical test is called the **null hypothesis (*H*0)**. The test is designed to assess the strength of the evidence *against* the null hypothesis. the null hypothesis is a statement of "no difference."

when conducting a test of significance, a **null hypothesis** is used. The term **null** is used because this hypothesis assumes that there is no difference between the two means or that the recorded difference is not significant.

Null Hypotheses denoted by H0.

The opposite of a null hypothesis is called the **alternative hypothesis**. The alternative hypothesis is the claim that researchers are actually trying to prove is true.
The claim about the population that evidence is being sought for is the **alternative hypothesis (*H*a)**.

❖ Test Statistic

- It is a random variable that is calculated from sample data and used in hypothesis test.
- Test statistic compare your data with what is expected under the null hypothesis.
- It is used to calculate P-Value.
- A test statistic measures the degree of agreement between a sample of the data and the null hypothesis.

Different hypothesis tests use different test statistics based on the probability model assumed in the null hypothesis. Common tests and their test statistics are:

Hypothesis Test	Test Statistics
Z-test	Z-statistic
T-test	T-statistic
ANOVA	F-statistic
Chi-square tests	Chi-square statistic

❖ P-Value

The ***p*-value** is the probability, computed under the assumption that the null hypothesis is true, of observing a value from the test statistic at least as extreme as the one that was actually observed.

Thus, P-value is the chance that the presence of difference is concluded when actually there is none.

- ❑ When the p value is between 0.05 and 0.01 the result is usually called significant.
- ❑ When P value is less than 0.01, result is often called highly significant.

- ❑ When p value is less than 0.001 and 0.005, result is taken as very highly significant.

❖ Statistical test

- These are intended to decide whether a hypothesis about distribution of one or more populations should be rejected or accepted.

Statistical Test

Parametric Test applied when data concern :

- Normal distribution
- Quantitative data
- Scale of measurement is METRIC scale(i.e. interval / ratio scale
- Compare mean and standard deviation
- More powerful

Non- Parametric Test applied when data concern :

- Skewed distribution
- Qualitative data
- Nominal or ordinal scale
- Compared in terms of %, proportion, etc.
- Comparatively less powerful

❑ Parametric tests

- Used for Quantitative Data
- Used for continuous variables
- Used when data are measured on approximate interval or ratio scales of measurements.
- Data should follow normal distribution.

✓ **Some parametric tests are:-**

- t-test
- ANOVA (Analysis of variance)
- Pearson’s r Correlation (r= rank)
- Z test for large samples(n>30)

❑ **Student's t- test**

Developed by Prof. W.S Gossett in 1908, who publishes statistical papers under the pen name of "student." Thus the test is known as Student's "t" test.

➢ When the test is apply?

1. When samples are small.
2. Population variance are not known.

✓ Assumption made in the use of "t" test

1. Samples are randomly selected.
2. Data utilized is Quantitative.
3. Variables follow normal distribution.
4. Sample variance are mostly same in both the groups under the study.
5. Samples are small, mostly lower than 30.

❑ Student’s t- test Cont…

- ✓ T- test compares the difference between two means of different groups to determine whether that difference is statistically significant.

- ✓ It is use in different – different purposes:
  - “t” test for one sample
  - “t” test for unpaired two samples.
  - “t” test for paired two samples.

✓ **One Sample t-test**

- When compare the mean of a single group of observations with a specified value.
- In one sample t-test, we know the population mean. We draw a random sample from the population and then compare the sample mean with the population mean and make a statistical decision as to whether or not the sample mean is different from the population.

<u>Formula :</u>

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Where, $\overline{x}$ = sample mean
μ= population mean, $\frac{s}{\sqrt{n}}$ = standard error.

Now we compare calculate value with table value at certain level of significance (generally 5% or 1%).

✓ One Sample t-test Cont…

- ✓ If absolute value of “t” obtained is grater than table value then reject the null hypothesis and if it is less than table value, the null hypothesis may be accepted.

Therefore, rule for rejecting the null hypothesis:

Reject Ho if $t \geq$ +ve Tabulated value

or,

Reject Ho if $t \leq$ -ve Tabulated value

or, we can say that $p < .05$

✓ T- test for unpaired two samples

- Used when the two independent random samples come from the normal populations having unknown or same variance.

- We test the null hypothesis that the two population means are same i.e., $\mu 1 = \mu 2$

- **Assumption made for use**

  1. Populations are distributed normally
  2. Samples are drawn independently and at random

- **When the test is apply?**

  1. Standard deviations in the populations are same and not known
  2. Size of the sample is small

✓ T- test for unpaired two samples Cont…

If two independent samples xi (i = 1,2,….,n1) and yj (j = 1,2, …..,n2) of sizes n1and n2 have been drawn from two normal populations with means μ1 and μ2 respectively.

Null hypothesis

H0 : μ1 = μ2

Under H0, the test statistic is

$$\boldsymbol{t} = \frac{|\boldsymbol{x} - \boldsymbol{y}|}{\mathbf{S\sqrt{1/n1 =+1/n2}}}$$

✓ T- test for paired two samples

Used when measurements are taken from the same subject before and after some manipulation or treatment.

Ex: To determine the significance of a difference in blood pressure before and after administration of an experimental pressure substance.

✓ T- test for paired two samples Cont…

- **Assumptions made for the test**

1. Populations are distributed normally
2. Samples are drawn independently and at random

- **When the test apply**

1. Samples are related with each other.
2. Sizes of the samples are small and equal.
3. Standard deviations in the populations are equal and not known.

Null Hypothesis:

H0: μd = 0

Under H0, the test statistic

$$t = \frac{|\bar{d}|}{s/\sqrt{n}}$$

Where, d = difference between x1 and x2

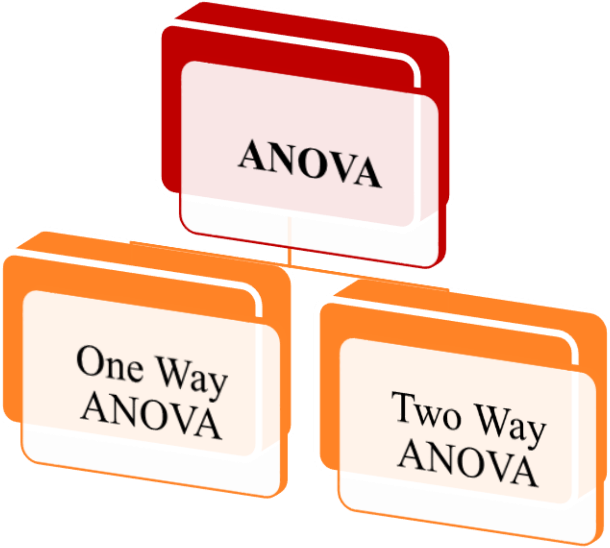
$\bar{d}$ = Average of d

s = Standard deviation

n = Sample size

❑ ANOVA (Analysis of Variance)

- Developed by R.A.Fischer.
- Analysis of Variance (ANOVA) is a collection of statistical models used to analyze the differences between group means or variances.
- Compares multiple groups at one time.



✓ One way ANOVA

Compares two or more unmatched groups when data are categorized in one factor.

Example :

1. Comparing a control group with three different doses of aspirin
2. Comparing the productivity of three or more employees based on working hours in a company

✓ Two way ANOVA

- Used to determine the effect of two nominal predictor variables on a continuous outcome variable.
- It analyses the effect of the independent variables on the expected outcome along with their relationship to the outcome itself.

Example :

Comparing the employee productivity based on the working hours and working conditions.

Assumptions of ANOVA :

- The samples are independent and selected randomly.
- Parent population from which samples are taken is of normal distribution.
- Various treatment and environmental effects are additive in nature.
- The experimental errors are distributed normally with mean zero and variance σ2

ANOVA compares variance by means of F-ratio

- F = $\mathit{variance\ between\ samples}$ / $\mathit{variance\ within\ samples}$
- It again depends on experimental designs

Null hypothesis:

Ho = All population means are same

- If the computed Fc is greater than F critical value, we are likely to reject the null hypothesis.
- If the computed Fc is lesser than the F critical value , then the null hypothesis is accepted.

❑ Z-test

ZTEST: The sample statistic is the sample mean of the data, $\bar{x}$. If **population standard deviation is known** (unlikely in real life), the distribution of the sample means is $\bar{X} \sim N\left(\mu_0, \frac{\sigma}{\sqrt{n}}\right)$, where μ_0 is the population mean *assumed in the null hypothesis*. The test statistic is a z-score: $z = \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$. The p-value is the tail area under the $\bar{X}$ normal curve beyond $\bar{x}$ in the direction of the alternative hypothesis, which is the same as the tail area under $Z \sim N(0,1)$ beyond z.

✓ Z test for large samples(n>30)

- Z-test is a statistical test where normal distribution is applied and is basically used for dealing with problems relating to large samples when the frequency is greater than or equal to 30.
- It is used when population standard deviation is known.

Assumptions made for use:

1. Population is normally distributed
2. The sample is drawn at random

When the test apply?

- Population standard deviation σ is known
- Size of the sample is large (say n > 30)

✓ Z test for large samples(n>30) Conti…

Let x1, x2, ………x , n be a random sample size of n from a normal population with mean μ and variance σ2 .

Let $\overline{x}$ be the sample mean of sample of size "n"

Null Hypothesis:

Population mean (μ) is equal to a specified value μo

$$H0: \mu = \mu o$$

Under Ho, the test statistic is

$$\boldsymbol{Z} = \frac{|\,\boldsymbol{x} - \mu \boldsymbol{o}|}{s/\sqrt{n}}$$

If the calculated value of Z < table value of Z at 5% level of significance, H0 is accepted and hence we conclude that there is no significant difference between the population mean and the one specified in H0 as μo.

❑ Non- Parametric Test

- ✓ **Non-parametric tests can be applied when:**
  - Data don’t follow any specific distribution and no assumptions about the population are made.
  - Data measured on any scale applied when data concern.

- ✓ **Commonly used Non Parametric Tests are:**

  1. Chi Square test
  2. Mann-Whitney U test
  3. Kruskal-wallis one-way ANOVA
  4. Friedman ANOVA
  5. The Spearman rank-order correlation test.

❑ Chi Square test

- First used by Karl Pearson
- Simplest & most widely used non-parametric test in statistical work.
- Calculated using the formula:- $\chi 2 = \Sigma (O – E)2 / E$

Where,
O = observed frequencies
E = expected frequencies

- Greater the discrepancy b/w observed & expected frequencies, greater shall be the value of $\chi 2$.

- Calculated value of $\chi 2$ is compared with table value of $\chi 2$ for given degrees of freedom.

❑ Chi Square test Cont…

✓ Application of chi-square test :

- Test of association (smoking & cancer, treatment & outcome of disease, vaccination & immunity).

- Test of proportions (compare frequencies of diabetics & non-diabetics in groups weighing 40-50kg, 50-60kg, 60-70kg & >70kg.).

- The chi-square for goodness of fit (determine if actual numbers are similar to the expected/theoretical numbers).

❑ Sources

1. These lecture notes are intended to be used with the open source textbook “Introductory Statistics" by Barbara Illowsky and Susan Dean (OpenStax College, 2013).
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THANK
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