#### महात्मा गाँधी केन्द्रीय विश्वविद्यालय MAHATMA GANDHI CENTRAL UNIVERSITY (Established by an Act of Parliament) Chanakya Parisar, Zila School Campus, Motihari District-East Champaran, Bihar - 845401



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# POSTGRADUATE PROGRAMME

## **Under Choice Based Credit System**

## M.A./M.Sc. in Mathematics

## (Effective for the students admitted in the Academic Year: 2023-24 onwards) Evaluation Criteria:

- 1. Mid-Semester Examination: 20%
- 2. End-Semester Examination: 60%
- 3. Comprehensive Continuous Internal Assessment (CCIA): 20%
  - a. Assignment/Class Test/Quiz/Presentation/Seminar: 15%
  - b. Attendance: 5%

## **Detailed Course Outline**

### **CORE COURSES**

Course Code: MATH4101

Course Title: Algebra I

Credits: 4 (total contact hr. 60)

**Course Objectives:** The objective of this course is to study group action, Sylow's theorems, simple groups, compositions series, solvable groups and Jordan-Holder theorem. The course also includes the study of various field extensions along with Galois theory.

### **Course Contents:**

**Unit I:** Class equation of finite groups, Cauchy's theorem, p-Groups, Group action, orbitstabilizer theorem, applications of group action: Generalized Cayley's theorem, index theorem. Sylow's theorems and their consequences, simple groups, simplicity of  $A_n$ , for  $n \ge 5$ .

**Unit II:** Normal series, composition series, solvable groups, nilpotent groups, Jordan-Holder theorem.

**Unit III:** Extension fields, finite and algebraic extensions, splitting fields, fundamental theorem of field theory, algebraic closure of fields.

**Unit IV:** Simple and normal extensions, separable and inseparable extensions, automorphism of field extension. Cyclotomic Polynomials and extensions, Galois Extensions, Fundamental theorem of Galois Theory.

### **Reference Books:**

1. Joseph A. Gallian, Contemporary Abstract Algebra 4<sup>th</sup> ed., Narosa Publishing House, New Delhi, 1999.

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- 2. I.S. Luthar, I.B.S. Passi, Algebra Volume 4: Field Theory, Narosa Publishing House, 2008.
- 3. D.S. Dummit and R.M. Foote, Abstract Algebra, 3<sup>rd</sup> ed., Wiley India Pvt. Ltd. 2011.
- 4. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd. New Delhi, 1975.
- 5. V.K. Khanna and S.K. Bhambri, A Course in Abstract Algebra, Vikas Publishing House, 2014.
- 6. N. S. Gopalakrishnan, University Algebra, 2<sup>nd</sup> ed., New Age International, 1986.
- 7. John B. Fraleigh, A First Course in Abstract Algebra, 7<sup>th</sup> ed., Pearson, 2003.
- 8. P. M. Cohn, Basic Algebra: Groups, Rings and Fields, Springer, 2005.
- 9. N. Jacobson, Basic Algebra, Volumes I & II, 2<sup>nd</sup> ed., Dover Publications, 2009.
- 10. T. W. Hungerford, Algebra, Springer-Verlag, 1981.

### Course Code: MATH4102

### Course Title: Real Analysis

### Credits: 4 (total contact hr. 60)

**Course Objectives:** The aim of this course is to introduce the concept of Riemann-Stieltjes integral, pointwise convergence and uniform convergence of sequence and series of functions, differentiation of vector-valued functions on  $\mathbb{R}^n$  and their properties, integration of functions over a rectangle in  $\mathbb{R}^n$ , change of variables, partition of unity.

### **Course Contents:**

**Unit I:** Review of least upper bound axiom and Riemann integration, definition and existence of Riemann-Stieltjes integral, properties of R-S integral, reduction of an R-S integral to a finite sum, mean value theorem for R-S integral, fundamental theorem of integral calculus.

**Unit II:** Sequence and series of functions, uniform convergence, uniform convergence and continuity, uniform convergence and integration, uniform convergence and differentiation, Weierstrass approximation theorem.

**Unit III:** Multivariable calculus: Differentiability of functions from  $\mathbb{R}^n$  to  $\mathbb{R}^n$ , partial derivatives and differentiability, directional derivatives, chain rule, mean value theorems, derivatives of higher-order, Taylor's formulas with integral reminder.

**Unit IV:** Inverse function theorem, implicit function theorem, partition of unity, change of variables.

#### **Reference Books:**

- 1. W. Rudin, Principles of Mathematical Analysis, 3<sup>rd</sup> ed., McGraw Hill, 1986.
- 2. T. M. Apostol: Mathematical Analysis; Narosa Publishing House, New Delhi, 1996.
- 3. M. Giaquinta and G. Modica, Mathematical Analysis: An Introduction to Functions of Several Variables, Birkhäuser, 2009.
- 4. S. C. Malik and Savita Arora, Mathematical Analysis, 2<sup>nd</sup> ed., New Age International Pvt. Ltd., 2005.

#### Course Code: MATH4103

Course Title: Topology

#### Credits: 4 (total contact hr. 60)

**Course Objectives:** The aim of this course is to introduce the theory of topological spaces which emphasises on those topics that are important to higher mathematics. The course focuses on the basic notions of topological spaces such as properties of continuous mappings,

compact and connected spaces, product spaces, separation axioms and basic theorems on topological spaces.

### **Course Contents:**

**Unit I:** Definition and examples of topological spaces, closed sets, closure, dense sets, neighbourhoods, interior, exterior and boundary points, accumulation point and derived set.

**Unit II:** Bases and sub-bases, subspace, continuous function and its characterization, homeomorphism, quotient space, projections and product topology.

**Unit III:** Countability and separation axioms, Lindelöf spaces, separable spaces, Urysohn's lemma, Metrizable spaces, Tietze extension theorem.

**Unit IV:** Compactness, basic properties of compactness, compactness and finite intersection property, local compactness, connected spaces and their basic properties, components, locally and path connected spaces, one-point compactification.

#### **References Books:**

- 1. J.L. Kelley, General Topology, Van Nostrand, 1995.
- 2. James R. Munkres, Topology, 2<sup>nd</sup> ed., Pearson International, 2000.
- 3. K.D. Joshi, Introduction to General Topology, 2<sup>nd</sup> ed., New Age International Private Limited, 2017.
- 4. J. Dugundji, Topology, 2<sup>nd</sup> ed., Allyn and Bacon, 2007.
- 5. George F. Simmons, Introduction to Topology and Modern Analysis, Indian Edition, McGraw-Hill Education, 2017.
- 6. N. Bourbaki, General Topology, Part I, Addison-Wesley, 1966.
- 7. S. Willard, General Topology, Addison-Wesley, 1970.
- 8. S.W. Davis, Topology, Tata McGraw Hill, 2006.

### Course Code: MATH4104

**Course Title:** Ordinary Differential Equations

### Credits: 4 (total contact hr. 60)

**Course Objectives:** The course emphasises on Picard's existence and uniqueness theorem, series solutions, Sturm-Liouville theorem, Green's function and nonlinear differential equations.

### Course Contents:

**Unit I:** Introduction, initial and boundary value problem, Picard's existence and uniqueness theorem, existence and uniqueness of initial value problem with examples, some special methods for solving second order linear differential equations.

**Unit II:** Simultaneous differential equations, geometrical interpretation, Pfaffian differential equations, Series solution: Ordinary and singular points, Frobenius method, Legendre and Bessel differential equations.

**Unit III:** Orthogonal set of functions and Sturm-Liouville problem, eigenvalues and eigen functions, orthogonality of eigen functions, oscillations and the Sturm separation theorem, Sturm comparison theorem, Green's function and its applications to boundary value problems.

**Unit IV:** System of linear differential equations, fundamental matrix, eigenvalue-eigenvector method, existence and uniqueness theorem and illustration.

### **Reference Books:**

- 1. E.A. Coddington, An Introduction to Ordinary Differential Equations, Prentice-Hall of India, Pvt. Ltd., New Delhi, 1989.
- 2. George F. Simmons and Steven G. Krantz, Differential Equations: Theory, Technique and Practice, McGraw Hill Higher Education, 2007.
- Steven G. Krantz, Differential Equations: Theory, Technique, and Practice with Boundary value problems, CRC Press, 3<sup>rd</sup> Edition, Taylor & Francis Group, Boca Raton London, New York, 2022
- 4. George F. Simmons, Differential Equations with Application and Historical Notes, 3<sup>rd</sup> Edition Taylor & Francis Group, Boca Raton London, New York, 2017.
- 5. Shepley L. Ross, Differential Equations, 3<sup>rd</sup> ed., Wiley publication, 2016.
- 6. W. E. Boyce and R.C. Diprima, Elementary Differential Equations and Boundary Value Problems, 9<sup>th</sup> ed., John Wiley and Sons, USA, 2009.
- 7. N. S. Koshlyakov, M. M. Smirnov and E. B. Gliner, Differential Equations of Mathematical Physics, North-Holland Publishing Company Amsterdam, 1964.
- 8. S. G. Deo, V Raghavendra, RasmitaKar, V Lakshmikantham, Textbook of Ordinary Differential Equations, McGraw Hill Education, 3<sup>rd</sup> ed., 2017.
- 9. M. Braun, Differential Equations and Their Applications, Springer-Verlag, New York, 4<sup>th</sup> ed., 2013.
- 10. Garrett Birkhoff and Gian-Carlo Rota, Ordinary Differential Equations, 4<sup>th</sup> ed., John Wiley & Sons, 1989.

### Course Code: MATH4105

Course Title: Operations Research

Credits: 4 (total contact hr. 60)

**Course Objectives:** This course introduces the techniques of Operations Research to solve linear programming problems, Game Theory and Inventory Control.

#### **Course Contents:**

**Unit I:** Linear programming problems, simplex method, revised simplex method, duality in linear programming problem, primal-dual solution relationship, duality theorems, dual simplex method.

**Unit II:** Game Theory: Two-person zero-sum games, the maximin-minimax principle, games with mixed strategies, graphical solutions of  $2 \times n$  and  $m \times 2$  games, dominance property, arithmetic method for  $n \times n$  games, general solution for  $m \times n$  rectangular games, limitations and extensions.

**Unit III:** Dynamic programming, characteristic of dynamic programming, Bellman's principle of optimality, solutions of discrete dynamic programming problem, applications of dynamic programming, solutions of L.P.P. by dynamic programming.

**Unit-IV**: Inventory control, Deterministic inventory problems with no shortages, deterministic inventory problems with shortages, EOQ problems with price breaks, multi-item deterministic problems.

### **Reference Books:**

- 1. F. S. Hiller and G. J. Leiberman, Introduction to Operations Research, 9<sup>th</sup> ed., McGraw-Hill International Edition, 2010.
- 2. G. Hadley, Nonlinear and Dynamic Programming, Addison Wesley, 1964.
- 3. H. A. Taha, Operations Research: An Introduction, 10th ed. Pearson Education Limited, 2017.
- 4. Kanti Swarup, P. K. Gupta and Man Mohan, Operations Research, Sultan Chand & Sons, New Delhi, 2014.
- 5. N. S. Kambo, Mathematical Programming Techniques, Revised Ed., Affiliated East-West Press Pvt. Ltd., New Delhi, 2008.

### Course Code: MATH4201

### Course Title: Algebra II

### Credits: 4 (total contact hr. 60)

**Course Objectives:** Objective of this course is to discuss polynomial rings and their properties and to find the solution of polynomial equations by radicals. It also emphasises on the decomposition theorem, canonical forms and quadratic forms for a linear transformation.

#### **Course Content:**

**Unit I:** Euclidean Domains, Principal Ideal Domains, Unique Factorization Domains. Polynomial Rings. Noetherian and Artinian rings, Solution of polynomial equations by radicals. Insolvability of the general equation of degree  $\geq$  5 by radicals.

**Unit II:** Similarity and diagonalizability of a linear transformation, Minimal polynomial of a linear transformation, Gershgorin theorem, invariant subspaces, cyclic subspaces, primary decomposition theorem.

**Unit III:** Canonical forms: Triangular form, Jordan and rational canonical forms, nilpotent transformation, index of nilpotency, invariants of a nilpotent transformation.

**Unit IV:** Bilinear forms, Symmetric and Skew-symmetric bilinear forms, Quadratic forms, canonical form of a real quadratic form, signature and index of a real quadratic form, Sylvester's law of inertia.

#### **References Books:**

- 1. S. Lang, Introduction to Linear Algebra, 2<sup>nd</sup> ed., Springer, 2005.
- 2. Gilbert Strang, Linear Algebra and its Applications, Thomson, 2007.
- 3. Kenneth Hoffman, Ray Kunze, Linear Algebra 2<sup>nd</sup> ed., PHI Pvt. Limited, 1971.
- 4. V.K. Khanna and S.K. Bhambri, A Course in Abstract Algebra, Vikas Publishing House, 2014.
- 5. Seymour Lipschutz and M. Lipson, Schaum's Outline of Linear Algebra, McGraw Hill Education, 3<sup>rd</sup> ed., 2017.

#### Course Code: MATH4202

**Course Title:** Differential Geometry of Curves and Surfaces

Credits: 4 (total contact hr. 60)

**Course Objectives** To introduce students to the local and global theory of curves and surfaces so that they can embark on further studies and research in topics like Algebraic Topology, Differential Topology, Riemannian Geometry, and allied areas.

### **Course Contents:**

**Unit I:** Curves in plane and space: parameterized curves, tangent vector, arc length, reparametrization, regular curves, curvature and torsion of smooth curves, Frenet-Serret formulae, arbitrary speed curves, Frenet approximation of a space curve, Osculating plane, osculating circle, osculating sphere, involutes, and evolutes, Isometries of  $\mathbb{R}^2$  and  $\mathbb{R}^3$ , Fundamental theorem of plane and space curves.

**Unit II:** Regular surfaces, Tangent, Normal and Orientability, generalized cylinder and generalized cone, Surfaces of revolution and Quadric surfaces, First fundamental form, Isometries of surfaces and conformal mapping of surfaces, equiareal maps and Theorem of Archimedes.

**Unit III:** Second fundamental form, Curvature of curves on a surface, Normal and Principal curvatures, Meusnier's theorem, Euler's theorem, Weingarten equations and Weingarten matrix, Geometric interpretation of principal curvatures, Umbilical points. Gaussian and Mean curvature.

**Unit IV:** Geodesics: Definition and basic properties, geodesic equations, geodesics on surfaces of revolution, Clairaut's theorem, geodesics as shortest paths, geodesic coordinates, Gauss's Theorem, Gauss equations, Codazzi-Mainardi equations, Gauss theorema Egregium, Gauss-Bonnet Theorem (statement only).

### Reference Books:

- 1. A. Pressley, Elementary Differential Geometry, Springer (Undergraduate Mathematics Series), 2001.
- 2. M. P. Do Carmo, Differential Geometry of Curves and Surfaces, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1976.
- 3. C. Bar, Elementary Differential Geometry, Cambridge University Press, 2001.
- 4. A. Gray, Differential Geometry of Curves and Surfaces with Mathematica, CRC Press, 1998.
- 5. C. E. Weatherburn, Differential Geometry three dimensions, vol-1, Cambridge University Press, 2016.
- 6. D. Somasundaram, Differential Geometry: A First Course, Alpha Science Int. Ltd., Harrow, U.K., 2005.

#### Course Code: MATH4203

Course Title: Complex Analysis

Credits: 4 (total contact hr. 60)

**Course Objectives:** The objective of this course is to study the fundamental ideas of complexvalued functions, analytic functions, conformal mapping, bilinear transformations, Cauchy's theorems, Cauchy's integral formulas, zeros and singularities, residue, contour integrations and analytic continuation of Riemann-Zeta function.

#### **Course Contents:**

**Unit I:** Elementary properties and examples of analytic functions, Harmonic Function, Laplace's equation, branch of logarithm, Cauchy's theorem, winding number, Cauchy's integral formula, Cauchy's Inequality, Liouville's theorem, fundamental theorem of algebra,

**Unit II:** Maximum modulus theorem, Taylor and Laurent series, zeros and singularities, classification of singularities, Weierstrass Theorem, identity theorem, Cauchy's Residue theorem, contour integration.

**Unit III:** Conformal mapping, bilinear transformations, Schwarz' lemma, Argument principle, Rouche's theorem, open mapping theorem,

**Unit IV:** Analytic continuation, power series methods, Riemann-Zeta function and its analytic continuation.

### **Reference Books:**

- 1. J.B. Conway, Functions of One Complex Variable, 2<sup>nd</sup> ed., Narosa, New Delhi, 1996.
- 2. L.V. Ahlfors, Complex Analysis, McGraw Hill Co., Indian Edition, 2017.
- 3. J. W. Brown and R. V. Churchill, Complex Variables and Applications, 8<sup>th</sup> ed., McGraw-Hill, 2014.
- 4. T.W. Gamelin, Complex Analysis, Springer, 2001.
- 5. L. Hahn, B. Epstein, Classical Complex Analysis, Jones and Bartlett, 1996.
- 6. D. C. Ullrich, Complex Made Simple, American Mathematical Society, 2008.
- 7. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publications, 2011.

#### Course Code: MATH4204

**Course Title:** Numerical Analysis

Credits: 4 (total contact hr. 60)

**Course Objectives:** The objective of this course is to provide a basic understanding of numerical methods to solve polynomial equation, non-linear equation, numerical integration, IVP, BVP and to improve the student's skill to solve problems of different areas.

#### **Course Contents:**

**Unit-I:** Polynomial equations-Descartes' rule of signs, root of a polynomial equation-iterative and direct methods, Rate of Convergence, solution of system of linear equations: error analysis for direct methods-Gauss elimination and Gauss-Jordan elimination method, convergence analysis for iterative methods- Gauss-Jacobi and Gauss-Seidel method.

**Unit-II:** Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation, methods based on undetermined coefficients, errors in numerical differentiation.

**Unit-III:** Numerical integration-methods based on interpolation and methods based on undetermined coefficients, Trapezoidal rule, Simpson rule, Gaussian quadrature formulae, Errors in quadrature formulae, Newton-Cotes formulae.

**Unit-IV:** Numerical solution of ordinary differential equations: initial value problems, existence and uniqueness of the solution of initial value problem, Taylor series, Picard's method, Euler's method, modified Euler method, Runge-Kutta methods, Numerical solutions of second-order boundary value problems

- 1. M.K. Jain, S.R.K. Iyenger and R.K. Jain, Numerical Methods for Scientific and Engineering Computations, New Age Publications, 2003.
- 2. M.K. Jain, Numerical Solution of Differential Equations, 4<sup>th</sup> ed., New Age, 2018.

- 3. S.S. Sastry, Introductory Methods of Numerical Analysis, 4<sup>th</sup> ed., Prentice-Hall of India, 2006.
- 4. D.V. Griffiths and I.M. Smith, Numerical Methods for Engineers, Oxford University Press, 1993.
- 5. C.F. Gerald and P.O. Wheatley, Applied Numerical Analysis, Addison- Wesley, 1998.
- 6. K.E. Atkinson, An Introduction to Numerical Analysis, 2<sup>nd</sup> ed., Wiley-India, 1989.
- 7. J.I. Buchaman and P.R. Turner, Numerical Methods and Analysis, McGraw-Hill, 1992.

### Course Code: MATH4205

**Course Title:** Partial Differential Equations

### Credits: 4 (total contact hr. 60)

**Course Objectives:** This course focuses on the formulation of the partial differential equations, solution of wave, heat and Laplace equations. It also covers maximum and minimum principles and Green's functions for elliptic, parabolic and hyperbolic equations.

### **Course Contents:**

**Unit I:** Classification of first order partial differential equations, Charpit's method, classification of second-order linear partial differential equations into hyperbolic, parabolic and elliptic PDEs, reduction to canonical forms, solution of linear homogeneous and non-homogeneous PDE, Monge's method.

**Unit II:** Derivation of wave, heat and Laplace equations, D'Alembert's and Riemann-Volterra solution of the one-dimensional wave equation, method of separation of variables and integral transforms.

**Unit III:** Cauchy-Kowalewskaya theorem, vibration of a finite string with fixed ends, spherical and cylindrical wave equation, eigenvalue problems and special functions, boundary value problems and applications, maximum and minimum principles, uniqueness and continuity theorems, higher-dimensional boundary value problems, Dirichlet problem for cylinder and sphere.

**Unit IV:** Laplace equation, elementary solutions of Laplace's equations, Kelvin's inversion theorem, theory of Green's function for Laplace's equation, a relation of Dirichlet's problem to the calculus of variations, Green's function for the two-dimensional equation.

- 1. L. C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, AMS, 1999.
- 2. K. Sankara Rao, Introduction to Partial Differential Equations, PHI Learning, 2010.
- 3. Robert C. Mcowen, Partial Differential Equations, Methods and Applications, Pearson Education Inc., 2003.
- 4. Fritz John, Partial Differential Equations, Springer-Verlag, 1986.
- 5. I.N. Sneddon, Elements of Partial Differential Equations, McGraw-Hill, 1988.
- 6. L. Debnath, Non-linear Partial Differential Equations for Scientists and Engineers, Birkhäuser, 2000.
- 7. Phoolan Prasad and Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, 1987.

8. T. Amarnath, An Elementary Course in Partial Differential Equations, 2<sup>nd</sup> ed., reprint Narosa, 2021.

#### Course Code: MATH4301

**Course Title:** Calculus of Variation and Integral equations

Credits: 4 (total contact hr. 60)

### **Course Objectives:**

The aim of this course is to introduce various methods to solve Volterra's and Fredholm's linear integral equations of second kind and develop skills to apply Integral equations in solving mathematical problems. Also develop the concept for solving variational problems with fixed boundaries, isoperimetric problems and variational problems with moving boundaries.

#### **Course Contents:**

**Unit I:** Classification of Integral equations, conversion of initial and boundary value problem into integral equations. Solution of Volterra's and Fredholm linear integral equations of second kind by method of successive approximation and iterative method.

**Unit II:** Solution of Fredholm integral equations of second kind with separable Kernels. Integral equations with symmetric kernels, eigen values, eigen functions, resolvent kernel, Application of integral equations.

**Unit III:** Variational problems with fixed boundaries, Euler-Lagrange equation, necessary and sufficient conditions for extremum, Euler-Ostrogradsky equation, isoperimetric problems and applications.

**Unit IV:** Variational problems with moving boundaries, transversality conditions, sufficient conditions for an extremum, direct methods in variational Problems.

#### **Reference Books:**

- 1. A.S. Gupta, Calculus of Variation with Applications, Prentice Hall of India, 2003.
- 2. Naveen Kumar, An Elementary Course of Variational Problems in Calculus, Narosa Publishing House, New Delhi, 2005.
- 3. M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall, Inc., 2000.
- 4. Bernard Dacorogna, Introduction to the Calculus of Variations, World Scientific, 2004.
- 5. Robert Weinstock, Calculus of Variations with Applications to Physics and Engineering, Dover Publications, 1974.
- 6. L. G. Chambers, Integral Equations, International Text Book Company Ltd., London, 1976.
- 7. F. G. Tricomi, Integral Equations, Interscience, New York, 1957.
- 8. R. P. Kanwal, Linear Integral Equation: Theory and Technique, Birkhauser, 1997
- 9. M. D. Raisinghania, Integral Equations and Boundary Value Problems, 2<sup>nd</sup>ed, S. Chand & Company Pvt. Ltd., New Delhi, 2016.

Course Code: MATH4302 Course Title: Classical Mechanics Credits: 4 (total contact hr. 60) **Course Objectives:** The aim of classical mechanics to understanding of the fundamental concepts in motion of rigid body, Euler's equations of motion, Generalized co-ordinates systems, Hamiltonian's principle for a conservative system and develop mathematical skills in formulating and solving physics problems.

### Course Contents:

**Unit I:** Moments and products of inertia, moment of inertia of a body about a line through the origin, Momental ellipsoid, rotation of co-ordinate axes, principal axes and principal moments, kinetic energy of rigid body rotating about a fixed point, angular momentum of a rigid body.

**Unit II:** Eulerian angle, angular velocity, kinetic energy and angular momentum in terms of Eulerian angle, Euler's equations of motion for a rigid body, rotating about a fixed point, torque-free motion of a symmetrical rigid body (rotational motion of Earth).

**Unit III:** Generalized co-ordinates systems, geometrical equations, Lagrange's equation for a simple system using D'Alembert principle, Deduction of equation of energy, deduction of Euler's dynamical equations from Lagrange's equations, Hamilton's equations, Ignorable co-ordinates, Routhian Function.

**Unit IV:** Hamiltonian's principle for a conservative system, principle of least action, Hamilton-Jacobi equation, Phase space and Liouville's Theorem, Canonical transformation and its properties, Lagrange and passion brackets, Poisson-Jacobi identity.

### **Reference Books:**

- 1. A. S. Ramsay, Dynamics Part II, CBS Publishers & Distributors, Delhi, 1985.
- 2. N. C. Rana and P.S. Joag, Classical Mechanics, Tata McGraw-Hill, 1991.
- 3. H. Goldstein, Classical Mechanics, Narosa, 1990.
- 4. J. L. Synge and B. A. Griffith, Principles of Mechanics, McGraw-Hill, 1991.
- 5. L. N. Hand and J. D. Finch, Analytical Mechanics, Cambridge University Press, 1998.
- 6. Naveen Kumar, Generalized Motion of Rigid Body, Narosa Publishing House, New Delhi, 2004.
- 7. P.V. Panat, Classical Mechanics, Narosa Publishing House, New Delhi, 2013.

#### Course Code: MATH4303

Course Title: Measure and Integration

Credits: 4 (total contact hr. 60)

**Course Objectives:** The objective of this course is to understand the concept of Lebesgue outer measure, measurable sets, measurable functions, Lebesgue integration, convergence of sequences of measurable functions and their integrals.

#### **Course Contents:**

**Unit I:** Countable and uncountable sets, cardinal numbers, Schroeder-Bernstein theorem, partially ordered sets, lattices, Zorn's lemma.

**Unit II:** Lebesgue outer measure, measurable sets,  $\sigma$ -algebra, Borel sets, theorems on measurable sets, non-measurable sets, The Cantor set and Cantor-Lebesgue function.

**Unit III:** Measurable function, properties of measurable functions, Egoroff's theorem, convergence in measure, simple function, simple approximation theorem.

**Unit IV:** Lebesgue integral of bounded measurable functions, comparison of Lebesgue and Riemann integral, bounded convergence theorem, Lebesgue integral of measurable non-negative functions, Fatou's lemma, monotone convergence theorem, general Lebesgue integral, Lebesgue dominated convergence theorem.

### **Reference Books:**

- 1. G. de Barra, Measure Theory and Integration, New Age International (P) Ltd., New Delhi, 2014.
- 2. P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, 3rd ed. , New Age International Publishers, 2019
- 3. H. L. Royden and P.M. Fitzpatrick, Real Analysis, 4th ed., Pearson, 2015.
- 4. Charles Swartz, Measure, Integration and Function spaces, World Scientific, 1994.
- 5. T. M. Apostol, Mathematical Analysis, , 2nd ed., Narosa Publishing House, New Delhi, 2002.

### Course Code: MATH4401

**Course Title:** Functional Analysis

### Credits: 4 (total contact hr. 60)

**Course Objectives:** To develop the concepts of normed linear spaces, Banach spaces, Hilbert spaces and the bounded linear operators on Hilbert spaces.

### **Course Contents:**

**Unit I:** Normed linear spaces, Banach spaces and examples, quotient space, bounded and continuous linear transformations, finite dimensional normed linear spaces, equivalent norms and characterizations, Riesz Lemma.

**Unit-II:** B(X, Y), dual of X, Bounded linear functionals, Hahn-Banach extension theorem and its applications, adjoint operator.

**Unit-III:** Open mapping theorem, closed graph theorem, uniform boundedness principle and its consequences.

**Unit-IV:** Hilbert space, Schwarz inequality, orthogonal complements, orthogonal sets, Examples of orthonormal basis in Hilbert spaces, Bessel's inequality, conjugate space, Riesz representation theorem, dual of a Hilbert space and reflexivity of Hilbert space.

- 1. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons, 2006.
- 2. Walter Rudin, Functional Analysis, Tata McGraw Hill, 2010.
- 3. G. Bachman and L. Narici, Functional Analysis, Dover Publications, 2000.
- 4. J. B. Conway, A Course in Functional Analysis, Springer, 2006
- 5. P.K. Jain, O.P. Ahuja and K. Ahmed, Functional Analysis, New Age International 2020.
- 6. B.V. Limaye, Functional Analysis, New Age International, 2017.
- 7. A. E. Taylor, Introduction to Functional Analysis, John Wiley, 1958.
- 8. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hills, 1963.

### Course Code: MATH4402

Course Title: Fluid Dynamics

Credits: 4 (total contact hr. 60)

**Course Objectives:** The main objective of this course is focused on to understand the motion of fluid and develop concept, models and techniques which enables to solve the problems of fluid flow and help in advanced studies and research in the broad area of fluid motion.

### **Course Contents:**

**Unit I:** Kinematics: Euler's and Lagrange's methods, Flux of a fluid, Equation of Continuity, Streamlines, Path lines and streak lines, Velocity potential, Irrotational and rotational motion, Vortex Lines, Equations of Motion-Lagrange's and Euler's equations of motion, Bernoulli's theorem, Boundary surfaces.

**Unit II:** General properties of Newtonian and Non-Newtonian and plastic fluids, Stress components in real fluid, Relations between rectangle components of stress, Connection between stresses and gradients of velocity, Navier-Stoke equations of motion, Plane Poiseuille and Couette flows between two parallel plates.

**Unit III:** Equations referred to moving axes, Impulse reactions, Stream function, Irrotational motion in two-dimensions, Complex velocity potential, Sources, Sinks, Doublets and their images, Conformal mapping, Milne-Thomson circle theorem.

**Unit IV:** Motion of circular and elliptic cylinders in an infinite mass of liquid, Kinetic energy of liquid, Theorem of Blasius, Motion of a sphere through a liquid at rest at infinity, Liquid streaming past a fixed sphere, Equation of motion of a sphere, Stoke's stream function, Vortex motion and its elementary properties.

- 1. W. H. Besant and A. S. Ramsey, A Treatise on Hydrodynamics, CBS Publishers and Distributors, Delhi, 1988.
- 2. W. H. Besant and A. S. Ramsey, Treatise on Hydromechanics, Part II, CBS Publisher, Delhi 1988.
- 3. S. W. Yuan, Foundations of Fluid Dynamics, Prentice-Hall of India, 1988.
- 4. G.K. Batchelor and Introduction to Fluid Mechanics, Foundation, Books, New Delhi 1991.
- 5. A. J. Chorin and A. Marsden, A Mathematical Introduction to Fluid Dynamics, Springer-Verlag, New York, 1993.
- 6. F. Chorlton, Text Book of Fluid Dynamics, CBS Publisher, 2005.
- 7. L.D. Landau and E.M. Lipschitz, Fluid mechanics, Pergamon Press, 1985.
- 8. R.W. Fox, P.J. Pritchard and A.T. McDonald, Introduction to Fluid Mechanics, 7<sup>th</sup> ed., John Wiley & Sons, 2009.
- 9. M.D. Raisinghania, Fluid Dynamics, 5<sup>th</sup> ed., S Chand & Company, 2003.

# ELECTIVE COURSES

## Elective Courses floated for 3<sup>rd</sup> Semester

#### Course Code: MATH4306

Course Title: Wavelet Analysis

Credits: 4 (total contact hr. 60)

**Course Objectives:** To study the Fourier transform on  $L^1(\mathbb{R})$  and  $L^2(\mathbb{R})$  spaces, some theorems related to Fourier transform, wavelets and wavelet transforms and its basic properties, Multi resolution Analysis, orthonormal wavelets and construction of wavelets.

#### **Course Contents:**

**Unit I:** Fourier Analysis: Fourier transforms in  $L^1(\mathbb{R})$ , Basic properties of Fourier transforms, Fourier transforms in  $L^2(\mathbb{R})$ , Poisson's Summation formula, the Shannon sampling theorem and Gibbs's phenomenon, Heisenberg's uncertainty principle.

**Unit II:** Definition and examples of wavelets, Continuous wavelet transforms and its basic Properties, continuous wavelet transform and Holder continuity.

Unit III: Discrete wavelet transforms, Frames and Frame operators, orthonormal wavelets.

**Unit IV:** Definition of Multi resolution Analysis and examples, properties of scaling functions and orthonormal wavelet Bases, construction of wavelets, cardinal B-splines, Franklin wavelet, Battle-Lemarie wavelet, Daubechies' wavelets.

#### **Reference Books:**

- 1. Lokenath Debnath and Firdous Ahmad Shah, Wavelet Transforms and their Applications, 2<sup>nd</sup> ed., Birkhauser, 2015.
- 2. Ingrid Daubechies, Ten Lectures on Wavelets, SIAM: Society for Industrial and Applied Mathematics, 1992.
- 3. C. K. Chui, An Introduction to Wavelets, Academic Press, 1992.
- 4. A. Boggess, and F. J. Narcowich, A First Course in Wavelets with Fourier Analysis, 2<sup>nd</sup> ed., Wiley, 2009.
- 5. Eugenio Hernandez and Guido L. Weiss, A First Course on Wavelets, CRC Press, 1996.
- 6. David F. Walnut, An Introduction to Wavelet Analysis, Birkhauser, 2004.
- 7. P. Wojtaszczyk, A Mathematical Introduction to Wavelet, Cambridge University Press, 1997.

#### Course Code: MATH4307

**Course Title:** Advanced Measure Theory

Credits: 4 (total contact hr. 60)

**Course Objectives:** To introduce measurable spaces, measurable functions, integration on abstract measurable spaces, signed measures, product measure, Baire sets, Baire measures, regularity of measures on locally compact spaces.

#### **Course Contents:**

**Unit I:**  $\sigma$ - ring and  $\sigma$ -algebra, measurable sets, measurable spaces, measurable functions, integration over abstract measure spaces.

**Unit II:** Signed measures, Hahn and Jordan decomposition theorems, Integration with respect to signed measures, Radon-Nikodym theorem, Lebesgue decomposition.

**Unit III:** L<sup>p</sup>-spaces, Riesz representation theorem for bounded linear functionals on L<sup>p</sup>-spaces, Product measures, Fubini's theorem, Tonelli's theorem.

**Unit IV:** Baire sets, Baire measure, continuous functions with compact support, regularity of measures on locally compact spaces, integration of continuous functions with compact support, Reisz-Markoff representation theorem.

### **Reference Books:**

- 1. H.L. Royden and P.M. Fitzpatrick, Real Analysis, 4<sup>th</sup> ed., Pearson, 2015.
- 2. P.R. Halmos, Measure Theory, Springer, 2014.
- 3. G. De Barra, Measure Theory and Integration, New Age International, 1981.
- 4. S. K. Berberian, Measure and Integration, AMS Chelsea Publications, 2011.
- 5. C. D. Aliprantis and O. Burkinshaw, Principles of Real Analysis, Academic Press, 2011.
- 6. M.E. Taylor, Measure Theory and Integration, American Mathematical Society, 2006.
- 7. D.L. Cohn, Measure Theory, Birkhauser, 1994.
- 8. Angus E. Taylor, General Theory of Functions and Integration, Dover Publications 2010.

### Course Code: MATH4308

**Course Title:** Theory of Optimization

Credits: 4 (total contact hr. 60)

**Course Objectives:** This course introduces the techniques of optimization for solving the reallife problems and obtaining the right solution by formulating, analysing and solving mathematical models.

#### **Course Contents:**

**Unit I:** Optimization in  $\mathbb{R}^n$  in parametric form, existence of solution, the Weierstrass Theorem, Unconstrained Optimization: Introduction, Gradient methods, Conjugate direction methods, Newton's method, Quasi Newton's method.

**Unit II:** Non-Linear constrained Optimization: Introduction, Lagrange's Theorem, Kuhn-Tucker Theorem, Wolfe's and Beale's algorithm for solving quadratic programming problems.

**Unit III:** Convexity and Optimization, Algorithm for Convex Optimization: Project Gradient Methods, Penalty Methods, Multi objective Programming, Goal programming.

**Unit IV:** Queueing Theory, Queueing System, Elements of Queueing System, Operating Characteristics of Queueing System, Probability Distributions in Queueing System, Classification of Queueing Models, Definition of Transient and Steady States, Poisson Queueing System.

- Edwin K. P. Chong, Stanislaw H. Zak, An Introduction to Optimization, 3<sup>rd</sup> ed., Johan Welly & Sons, Inc, 2003.
- David G. Luenberger and Yinyu Ye, Linear and Nonlinear Programming, 3<sup>rd</sup> ed., Springer, International Series in Operations Research and Management Science, 2008.
- 3. Andrzej Ruszczynski, Nonlinear Optimization, Princeton University Press, 2006.
- 4. S. S. Rao, Optimization Theory and applications, Halsted Press, 1984.
- 5. H. A. Taha, Operations Research: An Introduction, 10<sup>th</sup> ed., Pearson Education Limited, 2017.

### Course Code: MATH4309

Course Title: Mathematical Modelling

Credits: 4 (total contact hr. 60)

**Course Objectives:** The objective of the course is to introduce mathematical modelling, that is, the construction and analysis of mathematical models inspired by real life problems. The course will present several modelling techniques and the means to analyse the resulting systems.

### **Course Contents:**

**Unit I:** Simple situations requiring mathematical modelling, techniques of mathematical modelling, Classifications, Characteristics and limitations of mathematical models, Some simple illustrations, Mathematical modelling through differential equations, linear growth and decay models.

**Unit II:** Non-linear growth and decay models, Compartment models, Mathematical modelling in dynamics through ordinary differential equations of first order, Mathematical models through difference equations, some simple models, basic theory of linear difference equations with constant coefficients.

**Unit III:** Mathematical modelling through difference equations in economic and finance, Mathematical modelling through difference equations in population dynamic and genetics, Situations that can be modelled through graphs, Mathematical models in terms of Directed graphs.

**Unit IV:** Mathematical models in terms of signed graphs, Mathematical models in terms of weighted digraphs, Mathematical modelling through linear programming, Linear programming models in forest management, Transportation and assignment models.

#### **Reference Books:**

- 1. J. N. Kapur, Mathematical Modelling, 2<sup>nd</sup> ed., New Age Intern, Pub., 2021.
- 2. Lawrence Perko, Differential Equations and Dynamical Systems, Springer-Verlag, 2006.
- 3. Frank R. Giordano, William Price Fox, Maurice D. Weir, A First Course in Mathematical Modelling, 5<sup>th</sup> ed., Charlie Van Wagner, 2014.
- 4. Walter J. Meyer, Concept of Mathematical Modelling, McGraw-Hill, 1985.
- 5. F. Chorlton, Ordinary Differential and Difference Equations: Theory and Applications, Van Nostrand, 1965.
- 6. Sandip Banerjee, Mathematical Modelling: Models, Analysis and Applications, CRC Press, 2014.

#### Course Code: MATH4310

Course Title: Special Functions

Credits: 4 (total contact hr. 60)

**Course Objectives:** The aim of this course is to find the types of differential equations which solved by application of special function.

### **Course Contents:**

**Unit I:** Legendre Functions, Legendre polynomials, Recurrence relations for the Legendre polynomials, The formulae of Murphy and Roderigues, Series of Legendre polynomials,

Legendre's differential equation, Neumann's formula for the Legendre functions, Recurrence relations for the functions, Integral expression for the associated Legendre function, Surface spherical harmonics, Use of associated Legendre functions in wave mechanics.

**Unit II:** Gamma Function, Bessel Functions, The origin of Bessel functions, Recurrence relations for the Bessel coefficients, Series expansions for the Bessel coefficients, Integral expressions for the Bessel coefficients, Bessel's differential equation, Spherical Bessel functions, Integrals involving Bessel functions, The modified Bessel functions, Expansions in series of Bessel functions, The use of Bessel functions in potential theory, Asymptotic expansion of Bessel functions.

**Unit III:** The Functions of Hermite and Laguerre, the Hermite polynomials, Hermite's differential equation, the occurrence of Hermite functions in wave mechanics, The Laguerre polynomials, Laguerre's differential equation, The associated Laguerre polynomials and functions.

**Unit IV:** Hypergeometric Functions, The hypergeometric series, An integral formula for the hypergeometric series, The hypergeometric equation, Linear relations between the solutions of the hypergeometric equation, Relations of contiguity, The confluent hypergeometric function, Generalised hypergeometric series.

### **Reference Books:**

- 1. I. N. Sneddon, Special Functions of Mathematical Physics and Chemistry, 3rd ed., Longman Higher Education, 1980.
- 2. G. Andrews, R. Askey & R. Roy, Special Functions, Cambridge University Press, 2001.
- 3. L. Andrews, Special Functions for Engineers and Applied Scientists, 2<sup>nd</sup> ed., Oxford University Press, 1998.
- 4. N. N. Lebedev, Special Functions & Their Applications, Dover Publications Inc., 2003.
- 5. W. W. Bell, Special Functions for Scientists and Engineers, Dover Publications Inc., 2004.

### Course Code: MATH4311

Course Title: Fractional Calculus

Credits: 4 (total contact hr. 60)

**Course Objectives:** Fractional order calculus deals with integrals and derivatives of arbitrary non-integer order, and shares its origins with classical integral and differential calculus. The objective of this course is to discuss the use of fractional calculus in applied sciences and engineering.

#### **Course Contents:**

**Unit I:** The Iterated integral approach, differential equation approach, complex variable approach and Weyl transform, Fractional derivatives, Riemann-Liouville Fractional Integral, Laplace transform of Fractional Integral, Leibniz's formula for Fractional Integrals.

**Unit II:** A Class of functions, Leibniz's formula for Fractional Derivatives, Integral representations, Laplace transform of the fractional derivative, Fractional differential equations, linearly independent solutions, Solution of the homogeneous equations, Explicit representation of solution, Solution of the non-homogeneous Fractional differential equation. **Unit III:** Relation to the Green's function, Convolution of Fractional Green's functions, Reduction of Fractional differential equations, Semi-

differential equations, Fractional integral equations, Fractional differential equations with non-constant coefficients, Sequential Fractional differential equations, Some comparisons with ordinary differential equations.

**Unit IV:** A law of Exponents for Fractional integrals, Weyl Fractional derivative, Algebra of the Weyl transform, An application to ordinary differential equations, Abel's integral equation and the Tautochrone problem, Heaviside operational calculus and Fractional calculus, Potential theory and Liouville's problem, Fluid flow and the design of a Weir Notch.

### Reference Books:

- 1. Rudolf Hilfer, Application of Fractional Calculus in Physics, World Scientific Publishing Company, 2000.
- 2. Shantanu Das, Functional Fractional Calculus for System Identification and Controls, Springer, 2008.
- 3. Kenneth S. Miller, Bertram Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, 1<sup>st</sup> ed., Wiley, 1993.
- 4. Keith B. Oldham and Jerome Spanier, The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order, Academic Press, 1974.
- 5. J. Sabatier, O.P. Agrawal, J. A. Tenreiro Machado, Advances in Fractional Calculus, Springer, 2007.
- 6. Yuliya S. Mishura, Stochastic Calculus for Fractional Brownian Motion and Related Processes, Springer-Verlag, 2008.
- 7. Kenneth S. Miller, Bertram Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, Wiley, 1993.
- 8. B. Ross, Fractional Calculus and its Applications, Springer, 1975.
- 9. Vasily E. Tarasov, Fractional Dynamic: Applications of Fractional Calculus to Dynamics of Particles, Fields and Media, Springer, 2010.

### Course Code: MATH4312

Course Title: Advanced Algebra

Credits: 4 (total contact hr. 60)

**Course Objectives:** The course emphasizes the concepts of normal series, composition series and Zessenhaus lemma. Introduction of solvable groups, nilpotent group and fitting and Frattini subgroup will be studied and the students will be introduced to free group, presentation of a group and properties of a free group.

#### **Course Contents:**

Unit I: Zessenhaus lemma, Schreier's refinement theorem, Jordan-Holder theorem.

**Unit II:** Derived series, super solvable groups, minimal normal subgroup, Hall's theorem, Hall subgroup, p-complements, central series, Schur's theorem.

**Unit III:** Nilpotent groups, Fitting subgroup, Jacobi identity, Three subgroup lemma, Frattini subgroup, Burnside basis theorem. Indecomposable groups, Fitting's lemma, KrullSchmidt theorem, semidirect product.

**Unit IV:** Free group, Generators and relations of a group, Rank of free group, Projective and injective property of a free group, Free semigroup and representation of a quotient semigroup.

### **Reference Books:**

- 1. T.W. Hungerford, Algebra, Springer-Verlag, 1981.
- 2. D.J.S. Robinson, A Course in the Theory of Groups, Springer-Verlag, 1996.
- 3. J.S. Rose, A Course on Group Theory, Dover Publications, 2012.
- 4. J.J. Rotman, An Introduction to the Theory of Groups, Springer, 1995.
- 5. M. Suzuki, Group Theory-I, Springer, 2014.

#### Course Code: MATH4313

**Course Title:** Module Theory

Credits: 4 (total contact hr. 60)

**Course Objectives:** A module over a ring is a generalization of vector space over a field. The study of modules over a ring R provides us with an insight into the structure of R. Our aim is to realise the importance of rings and modules as central objects in algebra and to study some applications.

#### **Course Contents:**

**Unit I:** Basic concepts of Module theory, Quotient Modules and Module Homomorphisms, Generation of Modules, direct sum of modules, free modules.

**Unit II:** Exact sequences, split exact sequences, projective, injective and flat modules, dual basis, Baer's criterion, divisible modules.

**Unit III:** Tensor product of modules, chain conditions, Hilbert basis theorem.

Unit IV: Modules over Principal Ideal Domains, Semi Simple modules.

#### **Reference Books:**

- 1. M.F. Atiyah and I.G. MacDonald, Introduction to Commutative Algebra, CRC Press, Taylor & Francis, 2018.
- 2. P.M. Cohn, Classic Algebra, John Wiley & Sons Ltd., 2000.
- 3. P.M. Cohn, Basic Algebra: Groups, Rings and Fields, Springer, 2005.
- 4. D.S. Dummit and R.M. Foote, Abstract Algebra, 3<sup>rd</sup> ed., Wiley India Pvt. Ltd., 2011.
- 5. T.W. Hungerford, Algebra, Springer-Verlag, 1981.
- 6. N. Jacobson, Basic Algebra, Volumes I & II, Second Edition, Dover Publications, 2009.
- 7. I.S. Luthar, I.B.S. Passi, Algebra, Volume 3: Modules, Narosa Publishing House 2003.

#### Course Code: MATH4314

Course Title: Advanced Topology

**Credits:** 4 (total contact hr. 60)

**Course Objectives:** It is in continuation of Topology which covers some advanced topics of topology such as: Compactification, Metrizability, Uniform spaces and Function spaces.

#### Course Contents:

**Unit I:** One-point compactification, Stone-Céch compactification, Paracompact spaces, their properties and characterizations.

**Unit II:** Urysohn's lemma, Tietze extension theorem, Metrizable spaces and metrization theorems.

**Unit III:** Uniform spaces, weak uniformity, uniformizability, completion of uniform spaces.

**Unit IV:** Function spaces, Point-wise and uniform convergence, compact open topology, Stone Weierstrass theorem, Arzela-Ascoli's theorem.

### **Reference Books:**

- 1. S. Willard, General Topology, Dover Publications, 2004.
- 2. S. W. Davis, Topology, Tata McGraw Hill, 2006.
- 3. K. D. Joshi, Introduction to General Topology, 2nd ed., New Age International Private Limited, 2017.
- 4. J. Dugundji, Topology, McGraw-Hill Inc., US, 1988.
- 5. J. Munkres, Topology, 2nd ed., Pearson College Div., 2000.

### Course Code: MATH4315

Course Title: Number Theory

### Credits: 4 (total contact hr. 60)

**Course Objectives:** The aim of this course is to teach the basics of elementary number theory. Apart from teaching theory, the stress will be on solving problems.

### Course Contents:

**Unit I:** Divisibility, primes, division algorithm, greatest common divisor, Euclidean algorithm, fundamental theorem of arithmetic, perfect numbers, Mersenne primes and Fermat numbers. Arithmetical functions:  $\varphi(n), \mu(n), d(n), \sigma(n)$  their multiplicative property and evaluation, Mobious inversion formula and applications.

**Unit II:** Modular arithmetic, residue systems, Fermat's little theorem, Euler's generalization, Wilson's theorem, algebraic congruences: solution by inspection, counting mutually incongruent roots, about  $ax \equiv b \pmod{m}$ , Chinese remainder theorem, Application to cryptography.

**Unit III:** Quadratic congruence, congruence modulo powers of prime, primitive roots and indices, quadratic residues, Legendre and Jacobi symbols, Euler's criteria, Gauss's lemma about Legendre symbol, quadratic law of reciprocity.

**Unit IV:** Binary quadratic forms and an equivalence relation among them, reduced forms among binary quadratic forms, Farey sequence, definition and properties, approximation of an irrational number by Farey terms, representation of a positive integer as a sum of two and four squares, Diophantine equations: solutions of ax + by = c,  $x^2 + y^2 = z^2$ .

- 1. David M. Burton, Elementary Number Theory, Wm. C. Brown Publishers, Dubuque, Iowa 1989.
- 2. G.A. Jones and J.M. Jones, Elementary Number Theory, Springer-Verlag, 1998.
- 3. W. Sierpinski, Elementary Theory of Numbers, North-Holland, Ireland, 1988.
- 4. Niven, S.H. Zuckerman and L.H. Montgomery, An Introduction to the Theory of Numbers, John Wiley, 1991.
- Joseph H. Silverman, A Friendly Introduction to Number Theory, 4<sup>th</sup> ed., Pearson Education, 2019.
- 6. Thomas Koshy, Elementary Number Theory with Applications, 2<sup>nd</sup> ed., Academic Press, 2007.
- 7. T. Nagell, Introduction to Number Theory, AMS Chelsea Publishing, 2009.

8. G.H. Hardy and E.M. Wright, introduction to the theory of Numbers, 6<sup>th</sup> ed., Oxford University Press, 2008.

Course Code: MATH4316

Course Title: Coding Theory

Credits: 4 (total contact hr. 60)

**Course Objectives:** The theory of error-correcting codes uses concepts from algebra, number theory and probability to ensure accurate transmission of information through noisy communication links. The objective of this course is to introduce basic concepts of coding theory: Decoding and encoding, finite fields and linear codes, Hamming codes, parity checks.

#### **Course Contents:**

**Unit I:** Introduction to algebraic coding theory, Linear codes, Hamming weight, Generator matrix, Parity check matrix, Equivalence of linear codes, Encoding and decoding of linear codes, Syndrome decoding.

**Unit II:** Bounds on codes: Hamming bound, Perfect codes, Hamming codes and Golay codes, Gilbert bound, Gilbert-Varshamov bound for linear codes, Singleton bound and MDS codes.

New codes from old codes: Extending, puncturing, and shortening of codes, Subcodes, Direct sum construction, (u, u + v)-construction, Reed–Muller codes, Subfield codes.

**Unit III:** Basic concepts of finite fields, Cyclic codes, Cyclic codes as ideals, Generator polynomial of cyclic codes, Matrix representation of cyclic codes, Representation of cyclic codes by roots of unity, BCH bound.

**Unit IV:** Burst error correcting codes, Some special cyclic codes, BCH codes, Reed Solomon Codes, Decoding algorithm for Reed-Solomon codes.

#### **Reference Books:**

- 1. S. Ling and C. Xing, Coding Theory: A First Course, Cambridge University Press, 2004.
- 2. R. Hill, A First Course in Coding Theory, Oxford University Press, 1986.
- 3. W. C. Huffman and V. Pless, Fundamentals of Error Correcting Codes, Cambridge University Press, 2010.

#### Course Code: MATH4317

**Course Title:** Discrete Mathematics

Credits: 4 (total contact hr. 60)

**Course Objectives:** The main objective of this course is to evaluate Boolean functions and simplify expression using the properties of Boolean algebra; apply Boolean algebra to circuits and gating networks. Also, represent a graph using an adjacency list and an adjacency matrix and apply graph theory to application problems such as computer networks.

#### Course Contents:

**Unit I:** Partially and totally ordered sets (Review), Lattices, lattices as algebraic structures, sub-lattices, products and homomorphism, definition, examples and properties of modular and distributive lattices, complemented and complete lattices, Boolean algebra, Boolean polynomials, minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, switching circuits and applications of switching circuits.

**Unit II:** Complete Boolean Algebra, Boolean Rings, Sub algebras and generators, Boolean Homomorphisms, Ideals in Boolean Algebras and the fundamental theorem of homomorphism, the representation theorem for Boolean rings and Boolean Algebras, Boolean Space, the Stone representation theorem.

**Unit III:** Definition, examples and basic properties of graphs, hand-shaking lemma, complete graphs, bipartite graphs, isomorphism of graphs, paths and circuits, Eulerian circuits, Hamiltonian cycles, the adjacency matrix, Labeled and weighted graph, travelling salesman's problem, shortest path, Dijkstra's algorithm.

**Unit IV:** Planar graphs, Graph Coloring, Tree graphs and its application, Tree Traversal, Spanning Trees, Minimal Spanning Trees, Directed graphs, Rooted graphs, Sequential representation of Directed Graphs, Warshall's Algorithm; Shortest path.

### **Reference Books:**

- 1. R.P. Grimaldi, Discrete Mathematics and Combinatorial Mathematics, Pearson Education, 1998.
- 2. B A. Davey and H. A. Priestley, Introduction to Lattices and Order, Cambridge University Press, 1990.
- 3. Edgar G. Goodaire and Michael M. Parmenter, Discrete Mathematics with Graph Theory, Pearson Education (Singapore) Pvt. Ltd. Indian Reprint, 2003.
- 4. Kenneth H. Rosen, Discrete Mathematics and its Applications, McGraw Hill International, 2012.
- 5. C.L. Liu, Elements of Discrete Mathematics, McGraw Hill International Edition, 1986.

### Course Code: MATH4318

**Course Title:** Differentiable Manifolds

#### Credits: 4 (total contact hr. 60)

**Course Objectives:** To introduce students to the basics of differentiable manifolds so that they are able to appreciate better the topics covered in allied courses like Algebraic Topology, Differential Topology, Riemannian geometry and Riemann-Finsler geometry as well as be adequately prepared for pursuing research in these topics.

#### **Course Contents:**

**Unit I:** Tensors: summation convention and indicial notation, coordinate transformation and Jacobian, contravariant and covariant vectors, tensors of different type, algebra of tensors, contraction and covariant derivatives.

**Unit II:** Calculus in  $\mathbb{R}^n$ : tangent space. Vector fields, cotangent space and differentials on  $\mathbb{R}^n$ . Topological manifolds, Charts and atlases, differentiable manifolds, induced topology on manifolds, functions and maps, some special functions of class  $\mathbb{C}^{\infty}$ , para compact manifolds and partition of unity, pullback functions, local coordinates systems and partial derivatives.

**Unit III:** Tangent vectors and tangent space, differential of a map, the tangent bundle, pullback vector fields, Lie bracket, the cotangent space, the cotangent bundle, the dual of the differential map, one parameter group and vector fields.

**Unit IV:** Lie derivatives, tensors, tensor fields, connections, parallel translation, covariant differentiation of tensor fields, torsion tensor, curvature tensor, Bianchi and Ricci identities, geodesics, Riemannian manifolds.

### **Reference Books:**

- 1. F. Warner, Foundations of Differentiable Manifolds and Lie Groups, Springer, New York, 1983.
- 2. J. M. Lee, Introduction to Smooth Manifolds, GTM, Vol. 218, Springer, New York, 2003.
- 3. L. Conlon, Differentiable Manifolds, 2<sup>nd</sup> ed., Birkhauser Boston, Cambridge, 2001.
- 4. L. W. Tu, An Introduction to Manifolds, 2<sup>nd</sup> ed., Springer, 2011.
- 5. N. J. Hicks, Notes of Differential Geometry, D. Van Nostrand Reinhold Company, New York, 1965.
- 6. S. Kumaresan, A Course in Differential Geometry and Lie Groups (Texts and Readings in Mathematics), Hindustan Book Agency, 2002.
- 7. S. S. Chern, W. H. Chen and K. S. Lam, Lectures on Differential Geometry, World Scientific Publishing Co. Pvt. Ltd., 2000.
- 8. W. M. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, 2nd edition, Academic Press, New York, 2003.
- 9. R. Abraham, J. E. Marsden, T. Ratiu, Manifolds, Tensor Analysis and Applications, 2<sup>nd</sup> ed., Springer, 1988.
- 10. B. Spain, Tensor Calculus: A Concise Course, Dover Publications, 2003.

## Elective Courses floated for 4<sup>th</sup> Semester

#### Course Code: MATH4406

**Course Title:** Theory of Distributions

Credits: 4 (total contact hr. 60)

**Course Objectives:** To develop the concepts of Distributions and its basic properties, e.g., convergence of distributions, distributional derivative, Fourier transform of tempered distributions and Sobolev Spaces.

#### Course Contents:

**Unit I:** Test functions, Distributions, basic properties of distributions, convergence of distributions.

**Unit II:** Distributional Derivative, distributions of compact support, Convolution of distributions.

**Unit III:** The Space of rapidly decreasing functions, tempered distributions, Fourier transformation in S, Fourier transformation in S', convolution theorem in S', Fourier transformation in E', Applications to differential equations.

**Unit IV:** Sobolev space  $W^{\{m,p\}}$ , Sobolev space  $H^s$ , Product and convolution in  $H^s$ .

- 1. M.A. Al-Gwaiz, Theory of Distributions, CRC Press, 1992.
- 2. A. H. Zemanian, Distribution Theory and Transform Analysis: An Introduction to Generalized Functions with Applications, Dover Publications Inc, 2003.
- 3. A. H. Zemanian, Generalized Integral Transforms, Dover Publications Inc, 1987.
- 4. R. S. Pathak, A Course in Distribution Theory and Applications, Narosa, 2001.
- 5. R. Strichartz, A Guide to Distribution Theory and Fourier Transforms, World Scientific, 2003.

Course Code: MATH4407

**Course Title:** Automata Theory and Formal Languages

Credits: 4 (total contact hr. 60)

**Course Objectives:** The purpose of this course is to acquaint the student with an overview of the theoretical foundations of computer science from the perspective of formal languages.

### Course Contents:

**Unit I:** Theory of Computation: Finite automata, Deterministic and nondeterministic finite automata, equivalence of deterministic and non-deterministic automata.

**Unit II:** Moore and Mealy machines, Regular expressions, Grammars and Languages, Derivations, Language generated by a grammar.

**Unit III:** Regular Language and regular grammar, Regular and Context free grammar, Context sensitive grammar and Language, Pumping Lemma, Kleene's theorem.

**Unit IV:** Turing machines: Basic definitions, Turing machines as language acceptors, Universal Turing machines, decidability, undecidability, Turing machine halting problem.

### **Reference Books:**

- 1. D. Kelly, Automata and Formal Languages: An Introduction, Prentice-Hall, 1995.
- 2. J. E. Hopcroft, R. Motwani, and J. D. Ullman, Introduction to Automata, Languages, and Computation, 2nd ed., Pearson, 2001.
- 3. D. Kelly, Automata and Formal Languages: An Introduction, Prentice-Hall, 1995.
- 4. J. E. Hopcroft, R. Motwani, and J. D. Ullman, Introduction to Automata, Languages, and Computation, 2nd ed., Pearson, 2001.
- 5. P. Linz, An Introduction to Formal Languages and Automata, 6th ed., Jones and Bartlett Publishers, Inc, 2016.

### Course Code: MATH4408

Course Title: Operator Theory

### Credits: 4 (total contact hr. 60)

**Course Objectives:** To introduce eigenvalues and eigenvector of a linear operator, spectrum theory, compact linear operators, spectral properties of compact operators, spectral theory of self-adjoint operators and Banach Algebra.

#### **Course Contents:**

**Unit I:** Operators on Hilbert space, eigenvalues and eigenvector of a linear operator, spectrum of bounded operators, resolvent, spectral properties of bounded operators.

**Unit II:** Compact linear operators, Basic properties, adjoint of compact operators, Spectral properties of compact operators, Fredholm alternative.

**Unit III:** Spectral theory of self-adjoint operators, Spectral properties of self-adjoint operators, Positive operators and their properties, Spectral representation of a self-adjoint compact operator, Spectral family of a self-adjoint operator and its properties, Spectral representation of a self-adjoint operator, Continuous functions of self-adjoint operators.

**Unit IV:** Banach Algebra, Regular and Singular elements, Topological division of zero, spectral mapping theorem for polynomials and spectral radius formula, Ideals in Banach algebra, Commutative Banach algebra with examples, Gelfand transform, Maximal-ideal space with examples.

#### **Reference Books:**

- 1. Walter Rudin, Functional Analysis, Tata McGraw Hill, 2010.
- 2. G. Bachman and L. Narici, Functional analysis, Academic Press, New York, 1998.
- 3. B. V. Limaye, Functional Analysis, 3<sup>rd</sup> ed., New Age International Ltd., 2014.
- 4. J. B. Conway, A Course in Operator Theory: A Graduate Studies in Mathematics, Springer, 1985.
- 5. G. F. Simmons, An Introduction to Topology and Modern Analysis, Tata McGraw-Hill, 2004.
- 6. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons, India, 2006.
- 7. M. Schechter, Principles of Functional Analysis, 2<sup>nd</sup> ed., American Mathematical Society, 2001.
- 8. E. Kaniuth, A Course in Commutative Banach Algebras, Springer-Verlag, 2009.

#### Course Code: MATH4409

**Course Title:** Mathematical Statistics

Credits: 4 (total contact hr. 60)

**Course Objectives:** The main objective of this course is to provide students with the foundations of probabilistic and statistical analysis mostly used in varied applications in Engineering and Science.

#### **Course Contents:**

**Unit I:** Sample space, discrete probability, Probability inequalities (Tchebysheff, Markov, Jensen), independent events, Bayes theorem. Random variables and distribution functions (Univariate and multivariate), Hazard function and failure rates.

**Unit II:** Expectation and moments, Independent random variables, marginal and conditional distributions. Characteristic functions. Modes of convergence, weak and strong laws of large numbers, Central Limit theorem.

**Unit III:** Sampling, Methods of estimation, properties of estimators, confidence intervals, Analysis of discrete data and chi-square test, Tests of hypotheses: likelihood ratio tests.

**Unit IV:** Analysis of variance and covariance. Simple and multiple linear regression. Elementary regression diagnostics. Logistic regression, Multivariate normal distribution, Wishart distribution and their properties. Distribution of quadratic forms.

#### **Reference Books:**

- 1. Sheldon Ross, A First Course in Probability, 9th ed., Pearson Education India, 2013.
- 2. Robert V. Hogg, Joseph W. McKean and Allen T. Craig, Introduction to Mathematical Statistics, Pearson Education, 2007.
- 3. S.C. Gupta and V.K. Kapoor, Fundamentals of Mathematical Statistics, 12th ed., Sultan Chand & Sons, 2014.

Course Code: MATH4410 Course Title: Theory of Relativity Credits: 4 (total contact hr. 60) **Course Objectives:** This syllabus introduces the concept of special theory of relativity and its applications to Physical Sciences; and provides students with knowledge and proof of the validity of Physical Laws and to understand the concept of constant relative motion of different bodies in different frames of references.

### **Course Contents:**

**Unit I:** Frames of reference, speed of light and Galilean relativity, Michelson-Morley experiment, relative character of space and time, postulates of special theory of relativity, Lorentz transformation equations and its physical significance, Lorentz invariance

**Unit II:** Composition of parallel velocities, length contraction and time dilation, concept of simultaneity, Doppler's effect, relativistic velocity and acceleration transformation relations of a particle.

**Unit III:** Variation of mass with velocity, concept of zero rest mass of particle, mass energy relation, applications of mass-energy relation, binding energy, relativistic relation between mass-energy and momentum.

**Unit IV:** Four dimensional Minkowskian space, geometrical interpretation of Lorenz transformation, time-like, light-like and space-like intervals, world points and world lines, light cone, proper time, energy momentum four vectors, four vectors (world vectors), relativistic Lagrangian and Hamiltonian, relativistic equations of motion, Minkowski's Equation of motion.

### Reference Books:

- 1. P. G. Bergmann, Introduction to the Theory of Relativity, Prentice Hall of India, 1976.
- 2. R. Resnick, Introduction to special relativity, Wiley Eastern Pvt. Ltd, 2007.
- 3. W. Pauli, Theory of Relativity, Dover Publications, 1981.
- 4. Gregory L. Naber, The Geometry of Minkowski Space time: An Introduction to the Mathematics of the Special Theory of Relativity , Springer-Verlag, 2000.
- 5. David Bohm, The Special Theory of Relativity, Routledge, London and New York, 1996.
- 6. J. V. Narlikar, An Introduction to Special Theory of Relativity, Cambridge University Press, 2010.

### Course Code: MATH4411

Course Title: Finite Element Method

Credits: 4 (total contact hr. 60)

**Course Objectives:** This course aims to develop the knowledge to solve engineering problems. Use the finite element method to solve problems based on static and dynamic applications in mechanical systems.

#### **Course Contents:**

**Unit I:** Integral formulations and variational symbol initial and eigen value problems, functional, base functions, the variational symbol, formulation of boundary value problems.

**Unit II:** Plate bending elements, shell elements, interface elements, boundary elements, infinite elements, direct and variational formulations of element stiffness and loads, variational methods, variational methods of approximation: the Rayleigh-Ritz Method, Gelarkin's method of Weighted Residuals.

**Unit III:** Finite element analysis of one-dimensional problems, second-order and fourth-order boundary value problems and their applications, Eigen value and Time-dependent problems. **Unit IV:** Finite element analysis of two-dimensional problems, second-order equations for one scalar variable, torsion, heat transfer, boundary conditions and solution of overall problems, techniques of nonlinear analysis.

### **Reference Books:**

- 1. J. N. Reddy, Energy and variational Methods in Applied mechanics, Wiley, 2002.
- 2. O. C. Zienkiewicz, The Finite Element Method, Butterworth-Heinemann, 2000.
- 3. S. S. Sastry, Introductory Methods of Numerical Analysis, 4<sup>th</sup> ed., Prentice-Hall of India, 2006.
- 4. J. N. Reddy, An Introduction to the Finite Element Method, McGraw Hill, 2005.
- 5. N. Krishna Raju, Prestresses Concrete, 6<sup>th</sup> ed., Tata McGraw Hill Education, 2018.

### Course Code: MATH4412

**Course Title:** Fuzzy Sets and Applications

Credits: 4 (total contact hr. 60)

**Course Objectives:** This course introduces the Fuzzy sets and applications, geometric representation of Fuzzy sets, Fuzzy relations, Fuzzy logic and for solving the real life decision making problems where crisp formulation is not appropriate.

#### **Course Contents:**

**Unit I:** Basic Concepts of Fuzzy Sets: Motivation, Fuzzy sets and their representations, Membership functions and their designing, Types of Fuzzy sets, Operations on fuzzy sets, Convex fuzzy sets, Alpha-level cuts, Zadeh's extension principle, Geometric interpretation of fuzzy sets.

**Unit II:** Fuzzy Relations: Fuzzy relations, Projections and cylindrical extensions, Fuzzy equivalence relations, Fuzzy compatibility relations, Fuzzy ordering relations, Composition of fuzzy relations, Fuzzy Arithmetic: Fuzzy numbers, Arithmetic operations on fuzzy numbers.

**Unit III:** Fuzzy Logic: Fuzzy propositions, Fuzzy quantifiers, Linguistic variables, Fuzzy inference, Possibility Theory: Fuzzy measures, Possibility theory, Fuzzy sets and possibility theory, Possibility theory versus probability theory, Probability of a fuzzy event, Baye's theorem for fuzzy events, Probabilistic interpretation of fuzzy sets, Fuzzy mapping rules and fuzzy implication rules, Fuzzy rule-based models for function approximation, Types of fuzzy rule-based models (the Mamdani, TSK, and standard additive models).

**Unit IV:** Fuzzy Implications and Approximate Reasoning: Decision making in Fuzzy environment: Fuzzy Decisions, Fuzzy Linear programming, Fuzzy Multi criteria analysis, Multi-objective decision making.

- 1. J. Yen and R. Langari, Fuzzy Logic: Intelligence, Control and Information, Pearson Education, 2003.
- 2. G. J. Klir and B. Yuan, Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice-Hall of India, 1997.
- 3. H.J. Zimmermann, Fuzzy Set Theory and its Applications, Kluwer Academic Pub., 2001.

### Course Code: MATH4413

**Course Title:** Commutative Algebra

Credits: 4 (total contact hr. 60)

**Course Objectives:** To study commutative rings with unity and modules over the same that helps in developing basic foundation in other areas of mathematics such as algebraic geometry, homological algebra and algebraic number theory.

### **Course Contents:**

**Unit I:** Extension and contraction of ideals, Prime spectrum of rings, Jacobson radical of a ring, Prime avoidance lemma, Rings of formal power series, Restriction and extension of scalars.

**Unit II:** Localisation, Local properties, Extended and contracted ideals in rings of fractions, Primary decomposition, First and second uniqueness theorem of primary decomposition. **Unit III:** Integral dependence, Going up theorem, Going down theorem, Integrally closed domains, Valuation rings, Hilbert's Nullstellensatz theorem.

**Unit IV:** Noetherian rings, Primary decomposition in Noetherian rings, Artinian rings, Structure theorem for Artinian rings, Discrete valuation rings, Dedekind domains, Fractional ideals.

### **Reference Books:**

- 1. M.F. Atiyah and I.G. MacDonald, Introduction to Commutative Algebra, CRC Press, Taylor & Francis, 2018.
- 2. B. Singh, Basic Commutative Algebra, World Scientific, 2011.
- 3. D. Eisenbud, Commutative Algebra with a view towards Algebraic Geometry, Springer, 2004.
- 4. O. Zariski and P. Samuel, Commutative Algebra, Volume I & II, Springer, 1975.
- 5. R.Y. Sharp, Steps in Commutative Algebra, Cambridge University Press, 2000.

### Course Code: MATH4414

**Course Title:** Computational PDE

Credits: 4 (total contact hr. 60)

**Course Objectives:** This course exhibits the techniques for obtaining numerical solutions to partial differential equations and the basic ideas, theory behind those techniques and limitation of the technique.

#### **Course Contents:**

**Unit I:** Variational principle, approximate solutions of second order BVP of first kind by Reyleigh-Ritz, Galerkin, Collocation and finite difference methods, Finite Element methods for BVP-line segment, triangular and rectangular elements, Ritz and Galerkin approximation over an element, assembly of element equations and imposition of boundary conditions.

**Unit II:** Numerical solutions of parabolic equations of second order in one space variable with constant coefficients two and three levels explicit and implicit difference schemes, truncation errors and stability, Difference schemes for diffusion convection equation, Numerical solution of parabolic equations of second order in two space variable with constant coefficients-improved explicit schemes, Implicit methods, alternating direction implicit (ADI) methods.

**Unit III:** Numerical solution of hyperbolic equations of second order in one and two space variables with constant and variable coefficients-explicit and implicit methods, alternating direction implicit (ADI) methods.

**Unit IV:** Numerical solutions of elliptic equations, Solutions of Dirichlet, Neumann and mixed type problems with Laplace and Poisson equations in rectangular, circular and triangular regions, Finite element methods for Laplace, Poisson, heat flow and wave equations.

### **Reference Books:**

- 1. M. K. Jain, S. R. K. Iyenger and R. K. Jain, Computational Methods for Partial Differential Equations, Wiley Eastern, 1994.
- 2. M. K. Jain, Numerical Solution of Differential Equations, 2<sup>nd</sup> ed., Wiley Eastern, 2018.
- 3. S. S. Sastry, Introductory Methods of Numerical Analysis, Prentice-Hall of India, 2002.
- 4. D. V. Griffiths and I. M. Smith, Numerical Methods of Engineers, Oxford University Press, 1993.
- 5. C. F. General and P. O. Wheatley, Applied Numerical Analysis, Addison- Wesley, 1998.
- 6. K. J. Bathe and E. L. Wilson, Numerical Methods in Finite Element Analysis, Prentice-Hall, 1987.
- 7. A. S. Gupta, Text Book on Calculus of Variation, Prentice-Hall of India, 2002.
- 8. Naveen Kumar, An Elementary Course on Variational Problems in Calculus, Narosa, 2005.

### Course Code: MATH4415

Course Title: Riemannian Geometry

Credits: 4 (total contact hr. 60)

**Course Objectives:** Riemannian Geometry provide an important tool in modern mathematics, impacting on diverse areas from the pure to the applied. This course gives a thorough introduction to the theory of smooth manifolds with Riemannian metric. The course will provide a thorough understanding of Riemannian spaces and plications of Riemannian geometry to topology.

### Course Contents:

**Unit I:** Riemannian metrics, Riemannian manifolds, examples, Levi-Civita connection, fundamental theorem of Riemannian geometry, Curvature tensors- Riemannian curvature tensor, sectional curvature, Schur's Theorem, Ricci curvature, scalar curvature, Einstein manifolds.

**Unit II :** Gradient vector fields, divergence of a vector field, Covariant derivative along a curve, parallel transport, length of a curve. Distance function, geodesics, Exponential map, Jacobi fields, Gauss Lemma, complete Riemannian manifolds.

**Unit III:** Hopf –Rinow Theorem, The theorem of Hadamard, Riemannian immersions, second fundamental form, Gauss equation, Model spaces of constant curvature, Lie derivative, Lie derivatives of scalars, vectors, tensors and linear connections.

**Unit IV:** commutation formula for Lie differential operator and covariant differential operator, Motion, Affine motion, projective motion in a Riemannian space, curvature collineation, conformal and homothetic transformations.

#### **Reference Books:**

- 1. M. P. do Carmo; Riemannian Geometry, Berkhauser, 1992.
- 2. P. Peterson; Riemannian Geometry, Springer, 2006.
- 3. J. Jost; Riemannian Geometry and Geometric Analysis, Springer, 6<sup>th</sup> ed., 2011.
- 4. J. M. Lee; Riemannian Manifolds: An Introduction to Curvature, Springer, 1997.
- 5. S. Gallot, D. Hullin. J. Lafontaine; Riemannian Geometry, Springer, 3<sup>rd</sup> ed., 2004.
- 6. K. Yano; The Theory of Lie derivatives and its Applications, North Holland Publishing Company, Amsterdom, 1957.

#### Course Code: MATH4416

Course Title: Algebraic Topology

Credits: 4 (total contact hr. 60)

**Course Objectives:** Algebraic topology studies properties of topological spaces and maps between them by associating algebraic invariants (fundamental groups, homology groups, co-homology groups) to each space. The objective of this course is to understand some fundamental ideas in algebraic topology; to apply discrete, algebraic methods to solve topological problems; to develop some intuition for how algebraic topology relates to concrete topological problems.

#### **Course Contents:**

**Unit I:** Homotopic maps, homotopy type, retract and deformation retract.

**Unit II:** Fundamental group, Calculation of fundamental groups of n-sphere, cylinder, torus and punctured plane, Brouwer's fixed-point theorem, Fundamental theorem of Algebra.

Unit III: Free products, Free groups, Seifert-Van Kampen theorem and its applications.

**Unit IV:** Covering projections, Lifting theorems, Relations with the fundamental group, Universal covering space, Borsuk-Ulam theorem, Classification of covering spaces.

#### **Reference Books:**

- 1. G.E. Bredon, Geometry and Topology, Springer, 2014.
- 2. W. S. Massey, A Basic Course in Algebraic Topology, Graduate Texts in Mathematics 127, Springer-Verlag, 1991.
- 3. J.J. Rotman, An Introduction to Algebraic Topology, Springer, 2011.
- 4. T.B. Singh, Elements of Topology, CRC Press, Taylor & Francis, 2013.
- 5. E.H. Spanier, Algebraic Topology, Springer-Verlag, 1989.
- 6. A. Hatcher, Algebraic Topology, Cambridge University Press, 2002.
- 7. C. R. F. Maunder, Algebraic Topology, Cambridge University Press, 1980.
- 8. Satya Deo, Algebraic Topology, 2nd ed., Jainendra K Jain, 2018.

#### Course Code: MATH4417

Course Title: Cryptography and Network Security

Credits: 4 (total contact hr. 60)

**Course Objectives:** This course includes the basic theory of Cryptography and Network Security. The course is designed in such a way so that students will able to secure a message over insecure channel by various means and also understand various protocols for network security to protect against the threats in the networks.

### **Course Contents:**

**Unit I:** Definition of a cryptosystem, Symmetric cipher model, Classical encryption techniques-Substitution and transposition ciphers, Caesar cipher, Play fair cipher. Block cipher Principles, Shannon theory of diffusion and confusion, Data encryption standard (DES).

**Unit II:** Polynomial and modular arithmetic, Introduction to finite field of the form GF(p) and GF(2n), Fermat theorem and Euler's theorem (statement only), Chinese Remainder theorem, Discrete logarithm.

**Unit III:** Advanced Encryption Standard (AES), Stream ciphers, Introduction to public key cryptography, one-way functions, The discrete logarithm problem, Diffie-Hellman key exchange algorithm, RSA algorithm and security of RSA, The ElGamal public key cryptosystem, Introduction to elliptic curve cryptography.

**Unit IV**: Information/Computer Security: Basic security objectives, security attacks, security services, Network security model, Cryptographic Hash functions, Secure Hash algorithm, SHA-3. Digital signature, ElGamal signature, Digital signature standards, Digital signature algorithm.

### Reference Books:

- 1. William Stallings, Cryptography and Network Security, Principles and Practice, 5<sup>th</sup> ed., Pearson Education, 2012.
- 2. Douglas R. Stinson, Cryptography: Theory and Practice, CRC Press, 3<sup>rd</sup> ed., 2005.
- 3. J.A. Buchmann, Introduction to Cryptography, 2<sup>nd</sup> ed., Springer 2003.
- 4. W. Trappe and L.C. Washington, Introduction to Cryptography with Coding Theory, Pearson, 2006.
- 5. J. Hoffstein, J. Pipher, and J. H. Silverman, An Introduction to Mathematical Cryptography, 2<sup>nd</sup> ed., Springer, 2014.

### **Course Code: MATH4418**

Course Title: Theory of Elasticity

Credits: 4 (total contact hr. 60)

**Course Objectives:** The main objective of this course is to study the branch of Solid Mechanics which deals with the stress and displacements in elastic solids produced by external forces. The motive of study is to verify the sufficiency of the strength, stiffness and stability of structural and machine elements.

#### **Course Contents:**

#### Unit - I

Body and surface forces, Stress tensor, Equation of elasticity, Principal stress, Stress invariant, Normal stress, simple tension, Shear stress, Plane stress.

#### Unit - II

Deformation, Strain tensor, Geometric interpretation of components of strain, types of strain, Strain quadric of Cauchy, Principal axes, Principal strain, Strain invariant, Equation of compatibility.

#### Unit - III

Generalized Hooke's law, modulus of compression, Saint-Venant's principle, Mohr circle, curvilinear cylindrical and spherical coordinate.

### Unit - IV

Formulation of Extension, Torsion and Flexure of beams, Torsion solutions using Fourier methods, Euler-Bernauli's equation.

### **Reference Books:**

- 1. I. S. Sokolnikoff, Mathematical Theory of Elasticity, McGraw-Hill, 1946.
- 2. Martin H. Sadd, Elasticity: Theory, Application and Numerics, 2nd ed., Academic Press Inc, 2009.
- 3. L. D. Landau and E. M. Lifshitz, Theory of Elasticity (Course of Theoretical Physics), 2nd ed., Pergamon, 1981.
- 4. A. E. H. Love, A Treatise on the Mathematical Theory of Elasticity, Dover Publications Inc., 2003.
- 5. Y. B. Fu and R. W. Ogden, Nonlinear Elasticity: Application and Numerics, Cambridge University Press, 2001.

### Course Code: MATH4419

Course Title: Advanced Complex Analysis

Credits: 4 (total contact hr. 60)

**Course Objectives:** To understand the convex function and logarithmically convex function and its relation with maximum modulus theorem, the spaces of continuous, analytic and meromorphic functions, Runge's theorem and topics related with it, introduce harmonic function theory leading to Dirichlet's problem, theory of range of an entire function leading to Picard and related theorems.

#### **Course Contents:**

**Unit I:** Convex Functions and Hadamard's three circles theorem, Phragmen-Lindelöf theorem, Spaces of analytic functions, Hurwitz's theorem, Montel's theorem, Spaces of meromorphic functions.

**Unit II:** Riemann mapping theorem, Weierstrass' factorization theorem, Factorization of sine function, Runge's theorem, simply connected regions, Mittag-Leffler's theorem.

**Unit III:** Harmonic functions, Maximum and minimum principles, Harmonic function on a disk, Harnack's theorem, Sub-harmonic and super-harmonic functions, Maximum and minimumprinciples, Dirichlet's problems, Green's function.

**Unit IV:** Jensen's formula, Hadamard factorization theorem, Bloch's theorem, Picard theorems, Schottky's theorem.

- 1. J. B. Conway, Functions of One Complex Variable, 2<sup>nd</sup> ed., Narosa, New Delhi, 1996
- 2. L.V. Ahlfors, Complex Analysis, Mc. Graw Hill Co., Indian Edition, 2017.
- 3. T. W. Gamelin, Complex Analysis, Springer-Verlag, 2001.
- 4. L. Hahn, B. Epstein, Classical Complex Analysis, Jones and Bartlett, 1996.
- 5. W. Rudin, Real and Complex Analysis, 3<sup>rd</sup> ed., Tata McGraw-Hill, 2006.
- 6. E.C. Titchmarsh, The Theory of Functions, 2<sup>nd</sup> ed., Oxford University Press, 1976.

# **OPEN ELECTIVE COURSES**

Course Code: MATH4304

Course Title: MATLAB

Credits: 2 (total contact hr. 30)

**Course Objectives:** The objective of this course is to develop skill of students to solve geometric, computational and symbolic problems for different concepts in mathematics and to use technology appropriately to analyse mathematical problems.

### **Course Contents:**

**Unit I:** Introduction, Help Feature, MATLAB Windows, Types of Files, input and output, arithmetic, algebraic or symbolic computation, Some Useful MATLAB Commands.

**Unit II:** Scalars and Vectors, Entering data Matrices, Line Continuation, Matrix Subscripts/Indices, arithmetic operation between matrices, Generation of special matrices, solving linear system of equation.

**Unit III:** Entering polynomial, Polynomial evaluation, Roots of a polynomial, Polynomial addition and subtraction, polynomial multiplication, polynomial division.

**Unit IV:** Formulation of polynomial equation, characteristic polynomial of a matrix, polynomial differentiation, polynomial integration, Two-Dimensional Plots, Three-Dimensional Plots, figure windows, Sub plots, change of plot style.

### **Reference Books:**

- 1. Brian D. Hahn and Daniel T. Valentine, Essential MATLAB<sup>®</sup> for Engineers and Scientists, 3<sup>rd</sup>ed, Published by Elsevier Ltd, 2007.
- 2. Holly Moore, MATLAB<sup>®</sup> for Engineers, 3<sup>rd</sup> ed., Pearson, 2012.
- 3. Brian R. Hunt Ronald L. Lipsman Jonathan M. Rosenberg with Kevin R. Coombes, John E. Osborn Garrett J. Stuck, A Guide to MATLAB<sup>®</sup> for Beginners and Experienced Users, 2<sup>nd</sup> ed., Cambridge University Press, 2006.

#### Course Code: MATH4305

Course Title: LATEX

Credits: 2 (total contact hr. 30)

#### **Course Objectives:**

The objective of this course is to develop skill of students to have a working knowledge of the LATEX type setting language.

### COURSE CONTENTS:

#### Unit-I:

Installation of the software, LATEX distributions, environments, declaration, classfiles, Basic Syntax, packages, compilation, creating simple documents.

### Unit-II:

Alignments, array, mathematical commands, special characters and symbols, writing equations, equation numbering, matrix, tables.

### Unit-III:

Pictures inclusion, drawing with LATEX, plotting graph of real-valued functions of one variable.

### Unit-IV:

Page Layout: Length of page, page numbering, font style and size, line break, paragraph, colour, footnotes, margin, section, references/bibliography, labels and cross referencing, equation referencing, citations, writing resume, certificates,

### **Reference Books and online sources:**

- 1. Helmut Kopka and Patrick W. Daly, A guide to LATEX 2ε: Document Preparation for Beginners and Advanced Users, Addison-Wesley, 1995.
- 2. Michel Goossens, Frank Mittelbach and Alexander Samarin, The LATEX Companion, Addison-Wesley 1994.
- 3. The TEX Archive. <u>http://www.tex.ac.uk</u>

### Course Code: MATH4404

Course Title: Mathematica

### Credits: 2 (total contact hr. 30)

**Course Objectives:** This course provides a modern technical computing system programming language support the procedural, functional, object-oriented construct and parallel programming. The system is used in the many more technical, engineering, computing fields and mathematical.

#### **Course Contents:**

**Unit I:** The basic technique for using Mathematica, Adding text to notebooks, printing, creating slide shows and web pages, converting a notebook to another format, Mathematica's kernel, defining a function, plotting a function, Investigation functions and manipulate, Plotting implicitly defined functions, combining graphs,

**Unit II:** Managing data, importing data, difference equations, Factoring and expanding polynomials, finding roots of polynomials with Solve and DSolve, Solving equations and inequalities with reduce, working with rational functions and other expressions, Solving general equations and system of equations.

**Unit III:** Computing limits, derivative, higher order derivatives, maxima and minima, inflection points, implicit differentiation, differential equations, Integration, definite and improper integrals.

**Unit IV:** Matrices, matrix operations, minors and cofactors, working with large matrices, Solving systems of linear equations.

#### **Reference Books:**

- 1. Stephen Wolfram, The Mathematica Book, 4<sup>th</sup> ed., Cambridge University Press, 1999.
- 2. Cliff Hastings, Kelivin Mischo and Michael, Hands-on Start to Wolfram Mathematica: and Programming with Wolfram Language, 2<sup>nd</sup> ed., Wolfram Media, 2017.
- 3. Engene Don, Schaum's Outline of Mathematica, 2<sup>nd</sup> ed., McGraw-Hill Education, 2009.

Course Code: MATH4405 Course Title: SageMath Credits: 2 (total contact hr. 30)

### **Course Objective**

The main focus will be on using SageMath to explore topics in Calculus, Applied Linear Algebra and Numerical Method along with several applications.

### **Course Contents:**

**Unit I:** Introduction and Installation of SageMath, Exploring integers, Solving Equations, 2d and 3d Plotting, Calculus of one variable, Applications of derivatives, Integrals, Applications of Integrals,

**Unit II:** Partial Derivatives and gradients, Jacobians, Local maximum-minimum, Application of local maximum and minimum, Applications to least square problems, Lagrange Multipliers, Working with vectors, Solving system of linear Equations.

**Unit III:** Vector Spaces, Linear Transformations, Eigenvalues and Eigenvectors, Inner Product Spaces, Gram-Schmidt Process and QR-factorization, Singular Value Decomposition (SVD), Applications of linear algebra.

**Unit IV:** Numerical Solution of algebraic equations, Numerical Solutions of system linear equations, Interpolations, Numerical Integration, Numerical Eigenvalues, Solving ODE, Initial Value ODE, Solving system of ODE, Solving ODE with Laplace Transforms, Applications of ODE

#### **Reference Books and online sources:**

- 1. www.sagemath.org.
- 2. Mathematical Computation with Sage by Paul Zimmermann available from on <a href="http://www.sagemath.org">http://www.sagemath.org</a>
- 3. A First Course in Linear Algebra by Robert Beezer available online <u>http://linear.ups.edu/</u>.
- 4. Abstract Algebra: Theory and Applications by Tom Judson and Robert Beezer (<u>http://abstract.ups.edu/</u>).
- 5. Razvan A Mezei , An Introduction to SAGE Programming: With Applications to SAGE Interacts for Numerical Methods , Springer, 2016

\*\*\*\*\*The End\*\*\*\*\*