MAHATMA GANDHI CENTRAL UNIVERSITY

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- ECON3020
- ECONOMIC GROWTH
- COURSE CODE3020

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- THE RAMSEY CASS KOOPMANS MODEL

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The Ramsey-Cass-Koopmans model

- The RCK model is a finite overlapping model of economic growth in which the dynamics of economic aggregate are determined by decisions at the micro economic level (i.e by all consumers in the economy maximizing utility and all firms maximizing profit.

Assumption (about firms)

- All firms are identical.
- They hire worker and rent capital form a competitive market.
- Firms follow same production function as Solow followed to produce output Y=F(K,AL).
- Firms maximizes profits.
- Firms profit is household income.
- Firms take ‘A’ as given ,growing exogenously at ‘g’.

About Household

- There are large number of household in economy.
- The household grows at rate 'n'.
- K(O)/H is aggregate capital divided by number of households in the economy. Thus K(O)/H is capital per household.
- RCM model, unlike Solow model assumes no depreciation of capital.
- Household maximizes its utility(life time utility).
- To maximize life time utility, household divide income(from all sources) into consumption and saving.
- Sources of income from household are wages from labour, rent from capital, profit from firms.

House hold utility function

- The utility function for the representative household in the economy takes the form

$$U = \int_{t=0}^{\infty} e^{-\rho t} \mu(C_t) \frac{L_t}{H} dt$$

- Where, Ct is household consumption at time ‘t’.
- μ(.) is instantaneous utility.
- Lt is the total population of the economy.
- Lt/H is number of household.
- μ(Ct)Lt/H is therefore, household instantaneous utility.
- ρ is the discount rate.
- Higher discount rate means lower interest rate on future consumption. Therefore, increase in ρ decrease value of future consumption which means decrease interest eventually lower saving today.

The instantaneous utility function takes the form

- $\mu(C_t)=C_t^{1-\theta}/1-\theta$ $\theta>0$
- Where,
- θ is the rate of change in Marginal utility
- θ is coefficient of relative risk aversion.
- If we take derivative of $\mu(C_t)=C_t^{1-\theta}/1-\theta$ than,
- $\mu^{I}(.)=(1-\theta)C_t^{-\theta}/1-\theta=1/C_t^{\theta}$
- If $\theta=1$, consumption decrease $\Rightarrow$ Utility will increase.
- If $\theta=2$, Consumption decrease $\Rightarrow$ Utility will increase more rapidly.

Features of instantaneous Utility

- Increasing consumption will decrease Utility.
- If θ is high then household will not change current and future Consumption.
- $\rho - n - (1-\theta)g > 0$
- $\rho > n + (1-\theta)g$,,i,e Discount rate is higher than sum of Population growth rate.
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Maximzation behavior of firms ,firm behavior

- Firm produce output.
- Firm use inputs (labor & Capital).
- Firms pay labor and capital their marginal product which is their share of output.
- Firms earn zero profit which is normal profit because economy is competitive and production function is constant returns to scale.
- The marginal Product of capital is $\partial F(K,AL) / \partial K = f^{l}(K)$,which same as in solow model.
- As we assume no depreciation the real return on capital $(r_t) = f^{l}(k_t)$.
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Labour share

- Labour take the share as its marginal product.
- $Y=f(\,K, AL)$
- $Y/AL = f(K/AL)$
- $Y=ALf(K/AL)$
- Taking derivative with respect to L.
- $\partial Y/\partial L = ALf^1K(-K/AL^2)+f(K)A$
- Using product rule
- $A[f^1K\ \ -K/AL^2{}_X L]+f(K)A$
- $A[f^1K\ \ -K/AL+f(K)]$
- $A[f(K)\ \ -Kf^1\,(K)/AL] \qquad K=K/AL$ so]

Conti……

- $\partial Y/\partial L = A[f(k) - kf^{1}(k)]$
- This is actually marginal product of labor Thus wage earned by Labor at time 't' are
- $W_t = A_t[f(k_t - k_t f^{1}(k_t)]$
- Divide both side by 'A' to obtain 'w' effective 'L'.

Budget constraint of Houshold

- In our case, Income =Wage +Wealth Wage are present value of lifetime labor income and wealth is initial wealth.
- Consumption is the present value of lifetime consumption.

The household budget constraint will be

- Discounted lifetime consumption ≤Discounted lifetime resources.

$$\int_{t=0}^{\infty} e^{-R(t)} c_t \frac{L_t}{H} dt \leq \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} w_t \frac{L_t}{H} dt$$

- Where, K(o) is initial wealth can be written as

$$0 \leq \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} w_t \frac{L_t}{H} dt - \int_{t=0}^{\infty} e^{-R(t)} c_t \frac{L_t}{H} dt$$

- This means the difference is greater than zero and positive.

Household behavior

- To maximize the life utility, house hold has to choose path for consumption at each other, i.e. $C_{t.}$
- We can set a Hamiltonian optimization condition

$$\lambda = \beta \int_{t=0}^{\infty} e^{-\beta(t)} \frac{C_t^{1-\theta}}{1-\theta}\, dt + \lambda \left[K(0) + \int_{t=0}^{\infty} e^{-R(t)} e^{(n+g)t} \cdot w_t \, dt - \int_{t=0}^{\infty} e^{-R(t)} e^{(n+g)t} C_t \, dt \right]$$

- The household choose 'C' at each point of time. Thus, the first order condition for an individual, 'C_t' is

$$\frac{\partial \lambda}{\partial C_t} = \beta e^{-\beta(t)} \frac{1}{1-\theta} (1-\theta) C_t^{1-\theta-1} - \lambda e^{-R(t)} e^{(n+g)t} \cdot 1 = 0$$

Conti...

$$\Rightarrow \beta e^{-\beta t} c_t^{-\theta} = \lambda e^{-R(t)} e^{(n+g)t}$$

Taking logs on both sides

$$\ln\beta + -\beta t(\ln e) - \theta \ln c_t = \ln\lambda - R(t)\ln e + (n+g)t + \ln e$$

$$\text{Using, } R_t = \int_{t=0}^{t} r_t \, dt$$

$$\ln\beta - \beta t(\ln e) - \theta \ln c_t = \ln\lambda - \int_{t=0}^{t} r_t \, dt + (n+g)t$$

- Differentiate with respect to time

- $-\beta-\theta C_t^o/C_t=r_{(t)}+(n+g)$ $\quad$ $d\ln C_t/dt=C_t^o/C_t$
- $C_t^o/C_t=r_t-\rho-\theta g/\theta$ $\quad$ $\beta=\rho-n-(1-\theta)g$
- $C_t^o/C_t=r_t-\rho/\theta$ $\quad$ Capital C is consumption
- Per worker, small C is
- Consumption per unit of
- Effective labour

Conti

- This can be interpreted as, if real returns are more than discount rate of future consumption by household, consumption will increase.

Thanku………